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Mathematical Modelling of Surface-Groundwater Interactions under Varying Hydrological Conditions: Review of Past Works

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Authors' contributions

This work was carried out in collaboration among all authors. Author JCV designed the study, performed the literature survey, wrote the protocol and wrote the first draft of the manuscript. Author RKB managed the analyses of the study. Author VB helped in preparation of the manuscript. All authors read and approved the final manuscript.

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ABSTRACT

Surface-groundwater interaction is a research area of significant importance for its central role in wastewater treatment, irrigation, drainage, flood control, erosion and sediment control. Mathematical models are often used for the estimation of surface-groundwater interactions under the variety of hydrological conditions. Due to cost effectiveness and ability to accommodate variations in aquifer parameters, mathematical models have gained immense importance in the past few decades. The objective of this review paper is to portray the contribution of the hydrologist towards the growing area of surface-ground water interaction from all over the world who proposed, analyzed, executed and validated the developed Mathematical models. To begin with, we briefly introduce the main mathematical equations that govern the flow of groundwater in unconfined and confined aquifer systems. The development of stream-aquifer models is presented in a chronological order to provide a clear understanding of the contributions of past works. The methodology used in the past work is adequately discussed without going into mathematical

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details. Furthermore, we also summarize recent developments concerning groundwater flow in presence of vertical streambed, partial penetration, stream-stage variations and multiple recharge/discharge basins.

Keywords: Surface-groundwater; ditch-drain; sloping aquifer; water table; boussinesq equation.

1. INTRODUCTION

Surface-groundwater models are gaining immense importance in the past few decades due to applications in the assessment of baseflow, conjunctive management of groundwater resources, catchment hydrology, recharging and dewatering of aquifer and solute transport in the coastal aquifer system. Estimation of surface-groundwater interaction is also important in artificial recharge scheme, site remediation and irrigation system. Mathematical models have emerged as efficient and cost-effective tools to obtain the quantitative approximation of the interaction between groundwater resources and hydrologically connected water bodies. Analytical models are also useful in understanding the interplay between various aquifer parameters and hydraulic properties of the aquifer [1].

2. BRIEF HISTORY OF HYDROLOGICAL DEVELOPMENT

The relation between hydrostatic and hydrodynamic theory was well defined by physicists, mathematicians and scientist by the eighteenth century. Danial Bernoulli (1700-1782) was the first among them to show that in steady, incompressible inviscid flow the energy is conserved along a streamline. This is represented as:

$$\frac{V^2}{2g} + h + z = \text{constant} \tag{1}$$

Where *h* =depthofthefluid*, z* =potentialhead, $v^2/2g$= kinetichead.

In 1823, a differential relationship for velocity and pressure in unsteady and three-dimensional viscous flow was represented by Claude-Louis Navier (1785-1836). With the additional work, it resulted into the famous Navier-Stokes equation which serves as the mathematical basis for ideal fluid flow. Navier-Stokes equation forms the backbone of fluid dynamics and hydrology. The year 1860 can be remarked as the origin of the hydrological modelling. Later, Henry Darcy [2] performed an experiment of groundwater flow through sandy particles and established famous empirical formula known as Darcy's law which is a milestone in groundwater quantitative hydrology. Darcy's law states that the rate flow *Q*of water in a porous media is directly proportional to the cross sectional area *A* of the porous medium and piezometric head difference Q_1– Q_2between two points and inversely proportional to the length *L* of process media, i.e.

$$Q = K\frac{A(Q_1 - Q_2)}{L} \tag{2}$$

Where *K* is the hydraulic conductivity of the porous media.The basic equation of groundwater flow can be derived by applying the principle of mass conservation on a representative elementary volume. The resulting equation can be expressed as

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right) = S_s\frac{\partial h}{\partial t} \tag{3}$$

Where K_x, K_y, K_z are components of the hydraulic conductivity. *h* is the water head distribution and S_s is the specific yield.

Boussinesq [3] derived the nonlinear equation by combining Darcy's law with the mass balance equation which characterizes the governing equation of groundwater flow in most of the research pertaining to surface seepage flow. In order to accelerate the knowledge of groundwater flow mechanism, extensive innovations of mathematical modeling has been successively carried out for last one and half century. The earliest Mathematical model was developed in 1860. It was built on the basis of the Dupuit-Forchheimer hypothesis. In 1877 Bousinesq using method of separation of variable to obtain a general solution for governing equation of unconfined ground water flow on a bedrock in a plane with a steep hillslope. This mathematical approach was based on the assumption that the groundwater flow is almost parallel to the sloping bedrock. Theis [4] developed the relation between dropdown of piezometric head and the discharge rate and period of discharge from a well. This paper presents an analytical solution of an unsteady groundwater flow in a confined aquifer. This solution is based on Theis [4] function analogy. Subsequent developments in this field under category includes with and without storage from the confined aquifers. The research of Hantush and Jacob [5] throws light on the above fact. They obtained an analytical solution of transient radial ground water flow in confined leaky aquifer system under withdrawal of water from aquifer storage. This process is advanced with the finite or an infinite extent of considered domain.

An analytical solution of one dimensional groundwater flow for confined and homogeneous aquifer is developed by Hantush [6] Cooper and Rorabaug [7], Hall and Moench [8], Tolikas et al. [9]. Polubarinova-Kochina [10] developed a series solution of the Boussinesq equation for transient ground water flow. The innovations by Ibrahim and Brutsaert [11] and Verma and Brutsaert [12] assesses the reliability and limitations of Dupuit-Forchheimer approach. Wooding [13] developed an analytical solution for saturated flow over slanting bed employing conformal mapping technique. By applying extended Dupuit-Forchheimer assumption, the approximation solution for ground water flow over inclined impervious base is established by Child [14]. Marino [15] presented an analytical solution for the prediction of water table variations induced by uniform and time dependent recharge in ditch drain.Using vertical and horizontal axes for defining the governing ground water flow, a simplified version of Child [14] approximation is developed by Chapman [16]. Beven [17] used kinematic wave approach for deriving a solution for saturated and unsaturated flows. Zecharies and Brutseart [18] disproved the kinematic wave approach for aquifers possessing very small slopes.By applying Laplace transform method Verhoest et al. [19] presented the analytic solution of a transient ground water flow over a sloping unconfined aquifer which is applicable for time varying recharge rate as it exists in natural environment. By considering nonlinear equation, the mechanism of saturated subsurface flow over a hillslope of uniform width is analysed by many researcher Chapman [20] and Basha and Maalouf [21].

The approximation solution for the linearised version of Boussinesq equation is developed by Sanford et al. [22], Brutseart [23], Su [24] etc. Influence of varying recharge rate over water table fluctuations for distinct one dimensional and two dimensional flow is innovated by Singh and Rai [25], Rai and Singh [26-28], Rai et al. [29]. Effects of transient recharge over water table fluctuations in one dimensional sloping aquifer system is analysed by Ram and Chauhan [30], Singh et al. [31]. Innovations related to localised recharge over an inclined base is presented by Marino [32], Bansal [33]. An analytical solution of the water table fluctuations in two-dimensional aquifer system of finite extent is presented by Ramana et al. [34,35]. The prediction of water table variations inducedby a single recharge basin at constant rate over different geometrical shapes is presented in Marino [36] Marino [15] and Teloglou [37].

Mathematical model explaining growth of the water table induced by exponentially decaying recharge rate over circular shape unconfined aquifer subjected to Dirichiet boundary condition is depicted in Rai et al. [38]. Some studies present the snapshot of three-dimensional model in unconfined aquifer of finite and infinite thickness [39,40]. The three dimensional numerical model built on the basis of Richards equation and ADE (Advection Dispersion equation) to simulate the effect storm water penetrating basin over its aquifer is presented by Bahar et al. [41]. Butler and Zlontik [42] developed a solution for estimating withdrawal rate and stream depletion rate occurred due to pumping. The expression for the water head and flow rate of an inclined aquifer adjacent to the water bodies is presented by Zlontik and Haung [43], Upadhyay and Chauhan [44-46]. Bansal and Das [47-49], Verhoest et al. [12] developed analytical models for analysing stream aquifer interaction over sloping aquifer subjected to distinct hydrological constraints. The estimation of water head, flow rate and steady state profile of a groundwater flow in a homogeneous unconfined sloping aquifer subjected to seepage from adjacent streams of varying water level and constant recharge is presented by Vanikar and Bansal [50]. Singh and Jaiswal [51] presented a numerical solution of two-dimensional free flow of water subjected to time varying recharge to aquifersunderlain by a slanting impervious base. This work also depicts the picture of height dependent evapotranspiration impact on hydrological parameters.The unique model by considering Couchy's boundary condition and Robin boundary condition is developed by Teloglou and Bansal [52] and Moutsopoulos [53] subjected to different modes of water variations level.Numerical solution of the ground water flow by discretising spatial variable and temporal variable are obtained by using different techniques as finite difference method [54-57] and fourth order Runge-Kutta method [58,59]. Finite volume method is applied to obtain solution of two-dimensional unsaturated flow in regions of irregular shapes, for the prediction of two-dimensional transient ground water flow over unconfined slanting aquifer [60] to estimate depletion rates of a stream caused due to well pumping [61]. Seedpanah et al. [62] studies tidal fluctuations induced by ground water flow in coastal aquifers for studying ground water flow mechanism in porous medium of heterogeneous type.Three-dimensional analytical studies of the ground water flow is carried out by Haunget al. [63] and Chang et al. [64].

A digital numerical model was developed by Marino [65] applying predictor-corrector scheme for solving uni-dimensional Boussinesq equation in unconfined horizontal aquifer; however, Parlange [66] presented the solution by using partial differential equation solver PDE2D. The solution of one-dimensional Boussinesq equation of dimensionless format induced by constant recharge over unconfined sloping aquifer is derived by Beven [67]. Using finite difference method Renu et al. [68] developed numerical model for the dissolution of benzene and to study the transportation of aqueous state benzene in a fracture-matrix system of saturated nature subjected to steady-state ground water flow condition. The SUTRA [69], FEFLOW [70], MARUN [71] and SEWAT [72] are important benchmark of hydrology.Applying variational iteration method [73,74] developed analytical solution of nonlinear equation. A method of decomposition due to Adomian [75] proved to be efficient and accurate method for solving specially nonlinear ordinary and partial differential equations, integral equations, differential delay equations from engineering and applied sciences. The research work in this area is expanded by Wazwaz [76]. He developed new method, modified decomposition method. Duan [77] derived recurrence triangle formula for calculating polynomial. This work is followed by deduction of new algorithm for calculation of multivariable Adomian polynomials. The combination of these polynomials with other iterative techniques have been used for the approximation of nonlinear term of partial differential equations [78]. When these polynomials are accomplished by differential transform method via algebraic recurrence relation results in Taylor series solution however application of integral operator inversely with method selected from homotopy analysis method (HAM) results in series solution.

Attili [79] depicted the solution of Solitons by Adomian decomposition method.It is of great importance in the field of fluid dynamics, magneto-hydro dynamics and also in water wave studies. The solution of one-dimensional wave equation called as acoustic equation is presented by Dispini et al. [80] by using Adomian decomposition method. The classical study of ground water flow developed by sudden change in piezometric head of semi-infinite aquifer is presented by Moutsopoulus [81] by using Adomian Decomposition method. The numerical analysis of nonlinear ordinary differential equation of second order by using Adomian Decomposition method is presented by Agom et al. [82] where continued algorithm is implemented in discrete domain. This principal is adopted in Maple package. Next remarkable development is seen innewborn approaches and tools for analysing hydrological observations and data. Even though the development of these approaches cannot be traced in hydrology but can be adapted for applications in hydrology. Some of these approaches involve artificial neural networks [83,84], time series analysis [85], geostatistical approach [86], time series by employing FORTRAN95 [87], data assimilation approach, validation and calibration techniques [88]. The mathematical model based on Finite Differenc Techniques for forcasting the variations in the groundwater level and pressure water level is proposed by Daliev [89]. These tools and techniques helps in better understanding of hydrologic system and hydrologic processes. The above mentioned mathematical models widely cover different dimensional i.e.one, two,three dimensional flows and radial flow, distinct types of aquifer domains as finite, infinite and semi-infinite domain. These models are developed for different boundary condition as Dirichlet boundary condition, Robin boundary condition as well as no boundary conditions. By employing mathematical techniques and numericalprocedure, the solutions of these model are developed.In 1960s, with the introduction of computer and their amazing computational techniques and calculative power, the hydrological field gain a quantum leap in 1970s and 1980sand gave birth to digital hydrology or numerical hydrology. The history of hydrological development is flooded with girth of extensive work in this field however this review depicts the picture of few important developments occurred in last two to two and half decades. For easy reference these milestones of hydrology have been arranged theme wise.

3. CONCLUSIONS

The hydrological development of the groundwater flow in relation with the aquifer systems for last two centuries from first quarter of 19th century to first quarter of 21th centuryare reviewed. Numerous earlier studies reveal that the studies of the groundwater flow play a major and active role in developing the solutions pertaining to the arising problems from ecology, engineering, hydrogeology, engineering, and environmental areas. The initial period can be regarded as conceptual period where development is based on concept and relationship between parameters. The middle half of the 19^{th} century witnessed empirical approach which was necessity for establishing complex hydrological engineering projects as they were increasing number wise. This followed by flood estimating hydrological models. The birth of subsurface flow models was observed in the second half of 19 century. This period also witnessed the use of statistical methods to estimate the impact of storage on ground water flows as well as development of models using digital computer which is termed as digital hydrology or numerical hydrology. The later part of the above period shows the integration of hydrology with other branches such as ecohydrology, hydrogeology, coastal hydrology, hydro climatology, social hydrology etc. By considering the focus of above all aspects it's clear that groundwater is a multitasker.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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