Al Azhar Bulletin of Science Vol. (25) No.1, June, 15 - 20, 2014,.

PREDICTION OF THE ORBITAL MOTION OF AN ARTIFICIAL SATELLITE FROM RADAR MEASUREMENTS

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ABSTRACT

This paper presents the good prediction of motion an artificial satellite using radar data under perturbation forces, the perturbed J_4 Earth's gravity and the atmospheric drag with the best atmospheric density model which depends on both Sun magnetic field and solar activity.

To propagate the orbit, we have to determine the initial conditions by using at least three different values of range and their corresponding values of azimuth and elevation angles for radar data. The differential equations of satellite motion are solved using Runge-Kutta method of the fourth order with application on the radar data of EGYPTSAT-1.

INTRODUCTION

The simple configuration to determine the position and velocity of the satellite needs one ground station. The pointing angles in the topocentric system of the ground station are obtained by measuring the direction of the maximum signal amplitude of the satellite. The slant range or distance from the satellite to the station is computed from the round-trip time of a radar signal emitted from the ground station antenna to the satellite and radiated back to the station.

The range rate or line-of-sight velocity of the spacecraft relative to the ground station can be derived from the Doppler shift of a radar wave emitted from the ground station, transponded by the satellite, and received again at the ground station. (Oliver Montenbruck and Eberhard Gill, 2005)

2. Determination of the site's position vector

The first step is to determine the position vector of the ground station. The location of station gives from the following equation (Vallado, 2001)

$$\vec{r}_{osb} = \begin{bmatrix} r_i \cos(\theta) \\ r_i \sin(\theta) \\ r_k \end{bmatrix} \quad (2.1)$$

where

$$r_i = (C_\oplus + h)\cos(\theta) \quad (2.2.1)$$

$$r_k = (S_\oplus + h)\sin(\theta)\ , \quad (2.2.2)$$

$$S_\oplus = C_\oplus\,(1 - e_\oplus^2)\,, \quad (2.2.3)$$

$$C_\oplus = \frac{R_\oplus}{\sqrt{1 - e_\oplus^2 \sin^2(\theta)}}\ , \quad (2.2.4)$$

and also,

θ = local sidereal time,

$R_\oplus$ = the mean of equatorial radius of the Earth = 6378.1363 km,

$e_\oplus$ = the Earth's of eccentricity = 0.081819221456,

h = the height of station,

ϕ = the latitude.

3. Transformation from topocentric coordinate system (SEZ) to inertial coordinate system (IJK)

This transformation is based on at least three observations of slant range, azimuth and elevation (ρ_i, A_i and G_i where i =1, 2, 3). Since

$$\vec{\rho}_{SEZ} = \rho_S\,\vec{S} + \rho_E\,\vec{E} + \rho_Z\,\vec{Z} \quad (3.1)$$

where

$$\rho_S = -\rho\,\cos(El)\cos(Az), \quad (3.2.1)$$

$$\rho_E = -\rho\,\cos(El)\sin(Az), \quad (3.2.2)$$

$$\rho_Z = \rho\sin(Az)\ . \quad (3.2.3)$$

The azimuth angle (A) is measured clockwise from north; it takes values from 0° to 360°. While, the elevation angle (G) is measured from the horizontal to the radar line-of-sight; it takes values from -90° to 90°. The distance from ground station to the satellite is defined as, slant range (ρ).

Equation (3.1) is expressed in the top-centric coordinates system (SEZ). So, we have to convert $\vec{\rho}_{SEZ_i}$ to the inertial coordinate system (IJK).

The transformation matrix from topocentric coordinates system (SEZ) to the inertial coordinates system (IJK) is used. This transformation is achieved by two rotations. The first rotation is achieved through the local sidereal time θ; the second rotation is achieved through the latitude (Vallado, 2001). Thus, the transformation matrix is given by

$$T_{(SEZ\rightarrow ijk)_i} = \begin{bmatrix} (\sin(\phi)\cos(\theta))_i & (-\sin(\theta))_i & (\cos(\phi)\cos(\theta))_i \\ (\sin(\phi)\sin(\theta))_i & (\cos(\theta))_i & (\cos(\phi)\sin(\theta))_i \\ (-\cos(\phi))_i & 0 & (\sin(\phi))_i \end{bmatrix}, \quad i = 1, 2, 3. \quad (3.3)$$

Currently, the line-of-sight unit vector could be computed by the following relation

$$\hat{L}_i = T_{(SEZ\rightarrow ijk)_i} \begin{bmatrix} -\cos(G_i)\cos(A_i) \\ \cos(G_i)\sin(A_i) \\ \sin(G_i) \end{bmatrix}, \quad i = 1, 2, 3. \quad (3.4)$$

From equations (2.1, 3.1 and 3.4) we deduce that

$$\vec{r}_{(ijk)_i} = \text{r}_i \hat{L}_i + \vec{r}_{(osb)_i}, \quad i = 1, 2, 3. \quad (3.5)$$

The last equation gives three positions vectors $(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ in the inertial coordinate system.

Now we briefly discuss the Gibbs Method to get $\vec{v}_2$ which is corresponding to $\vec{r}_2$ (Vallado, 2001 and Bate et al., 1971) as follows. The velocity $\vec{v}_2$ can be written as

$$\vec{v}_2 = \frac{L_g}{r_2}\vec{B} + L_g \vec{S}, \quad (3.6)$$

where

$$\vec{B} \equiv \vec{D} \times \vec{r}_2 \quad (3.7.1)$$

$$L_g = \sqrt{\frac{\mu}{N D}}, \quad (3.7.2)$$

$$\vec{S} = (r_2 - r_3)\vec{r}_1 + (r_3 - r_1)\vec{r}_2 + (r_1 - r_2)\vec{r}_3, \quad (3.7.3)$$

$$\vec{D} = (\vec{r}_1 \times \vec{r}_2) + (\vec{r}_2 \times \vec{r}_3) + (\vec{r}_3 \times \vec{r}_1), \quad (3.7.4)$$

$$\vec{N} = |r_1|(\vec{r}_2 \times \vec{r}_3) + |r_2|(\vec{r}_3 \times \vec{r}_1) + |r_3|(\vec{r}_1 \times \vec{r}_2), \quad (3.7.5)$$

4. Perturbations Forces

This section is considered with the selected perturbations force as mentioned above.

4.1 Earth's gravity

The Earth is not a perfect sphere, it has an eggplant shape. The effects of Earth's oblateness are gravitational differences or perturbations. These effects are significant in low and medium orbit.

If $\vec{x}$ is the position vector of the satellite in the inertial frame, the equations of motion will be described bv

$$\ddot{\vec{x}} + \frac{\mu}{r^3}\vec{x} = -\frac{\partial V}{\partial \vec{x}} + \vec{P}^*, \quad (3.8)$$

where

- μ is the Earth's gravitational constant,
- r is the distance of the satellite from the origin, since

$$r^2 = x_1^2 + x_2^2 + x_3^2,$$

- V is the perturbed time-independent potential, given by

$$V = \frac{3}{2}\mu R_\oplus J_2 r^{-5} x_3^2 - \frac{1}{2}\mu R_\oplus J_2 r^{-3} + \frac{5}{2}\mu R_\oplus J_3 r^{-7} x_3^3 - \frac{3}{2}\mu R_\oplus J_3 r^{-5} x_3 + \frac{5}{8}\mu R_\oplus J_4 r^{-9} x_3^4 - \frac{30}{8}\mu R_\oplus J_4 r^{-7} x_3^2 + \frac{3}{8}\mu R_\oplus J_4 r^{-5}, \quad (3.9)$$

- $\vec{P}^*$ is the resultant of all non-conservative perturbing forces and forces derivable from a time-dependent potential. Consequently, $\vec{P}^*$ depends on several forces. In the present work, as we mentioned above $\vec{P}^*$ consists of the drag force only.

Now we have to determine the accelerations due to Earth's gravitational only or (J_2, J_3 and J_4) as the accelerations due to J_2 is

$$\begin{bmatrix} \vec{a}_x \\ \vec{a}_y \\ \vec{a}_z \end{bmatrix}_{J_2} = -\frac{3\mu R_\oplus^2 J_2}{2r^5} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 1 - \frac{5x_3^2}{r^2} \\ 1 - \frac{5x_3^2}{r^2} \\ 3 - \frac{5x_3^2}{r^2} \end{bmatrix} \quad (3.10)$$

the accelerations due to J_3 is

$$\begin{bmatrix} \vec{a}_x \\ \vec{a}_y \\ \vec{a}_z \end{bmatrix}_{J_3} = -\frac{5\mu R_\oplus^3 J_3}{2r^7} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 3x_3 - \frac{7x_3^3}{r^2} \\ 3x_3 - \frac{7x_3^3}{r^2} \\ 6x_3 - \frac{7x_3^3}{r^2} \end{bmatrix} \quad (3.11)$$

- and finally the accelerations due to J_4 is

$$\begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix}_{J_4} = \frac{15\mu R_\oplus^4 J_4}{8r^7}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\begin{bmatrix}1-\dfrac{14x_3^2}{r^2}+\dfrac{21x_3^4}{r^4}\\ 1-\dfrac{14x_3^2}{r^2}+\dfrac{21x_3^4}{r^4}\\ 5-\dfrac{70x_3^2}{3r^2}+\dfrac{21x_3^4}{r^4}\end{bmatrix} \quad (3.12)$$

4.2 The atmospheric drag

The drag force depends on the satellite's coefficient of drag and its velocity. This differs widely among satellites. The resistance of the atmosphere is one of the most important perturbing forces in the altitude region from (200-600) Km, then the drag force per unit mass of the satellite can be represented (Kampos, 1968 and Filzpatrick, 1970) by

$$\vec{a}_{drag} = -\frac{1}{2}\frac{C_d A}{m}\rho v_{rel}^2 \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|}, \quad (3.13)$$

where

- C_d is the non – dimensional drag coefficient (2.0 to 2.2),
- A is the cross-sectional area of the satellite,
- m is the mass of the vehicle,
- The ratio $\frac{C_d A}{m}$ is call ballistic coefficient (BC),
- $\vec{v}_{rel}$ is velocity vector is relative to the atmosphere.
- ρ is the atmosphere density,

This density has many irregular and complex variations both in time and position. It is largely affected by solar activity and by the heating or cooling of the atmosphere. The time variations are difficult to be included in an analytical expression. Since the atmosphere is not actually spherically symmetric but tends to be oblate, we have to count for these oblations in any expression for the density. There are some important factors that affect the atmospheric density: Diurnal variations, Solar-rotation, Sun spots, Magnetic-storm variations and etc.

In this paper, using density modal called GOST model atmosphere (ГОСТ 25645.115-84, 1991). This model is developed empirically from observation of the orbital motion of Russian creation satellites. The model includes the dependence of the density on solar and geomagnetic activity as well as the diurnal and semiannual density variation. This model is valid for satellites in the range of 120-1500 km.

Now we have to determine the accelerations due to the atmospheric drag only as

$$\begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix}_{drag} = \frac{-\frac{1}{2}\frac{C_d A}{m}\rho v_{rel}^2}{|\vec{v}_{rel}|}\begin{bmatrix}\dot{x}+\omega_\oplus y\\ \dot{y}-\omega_\oplus x\\ \dot{z}\end{bmatrix}, \quad (3.14)$$

where $\omega_\oplus$ is the west-to-east angular velocity of the atmosphere.

5. RESULTS AND DISCUSSION

Finally the equation of motion of satellite under the select perturbation force can be written as

$$\begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix} = -\frac{\mu}{r^3}\begin{bmatrix}r_x\\ r_y\\ r_z\end{bmatrix} + \begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix}_{J_2} + \begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix}_{J_3} + \begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix}_{J_4} + \begin{bmatrix}\vec{a}_x\\ \vec{a}_y\\ \vec{a}_z\end{bmatrix}_{drag}. \quad (5.1)$$

We'll use the Runge-Kutta method of the fourth order to solve numerically the above equation (the differential equations of satellite motion under perturbation).

Now, let us consider as a real example the radar data of EGYPTSAT-1, which has mass 160 Kg and ballistic coefficient 0.002 m^2/Kg, $W_\oplus$ = 7.292115833×10^{-5} rad/sec (Awad, 1988), ground station coordinates (ϕ = 30°.0503, λ = 31°.6070 and h = 340.7664 m) and the radar data are

i	Date and time	Ai (Deg.)	Gi (Deg.)	ρi (Km)
1	2011/04/20 06:54:46.098	48.760	0.000	2998.225071
2	2011/04/20 06:56:45.344	64.707	4.229	2560.9551
3	2011/04/20 06:58:45.344	85.502	6.679	2339.53135

Km

From the transformation matrix (Eq.3.5) we can get

$$\begin{bmatrix}r_1\\ r_2\\ r_3\end{bmatrix} = \begin{bmatrix}4992.45412139538 & 856.280552507694 & 4885.33536701409\\ 5582.50243508205 & 783.171393977608 & 4214.00016261085\\ 6084.86034218512 & 696.783219066428 & 3469.2612846468\end{bmatrix}$$

Using the Gibbs Method (Eq. 3.6 and 3.7) we get

$$v_2 = [4.58102142046041i \quad -0.6681608152j \quad -5.9333526114k]$$

Km/sec

So, from r_2 and v_2 we can calculate orbital elements (initial conditions) which are

a = 7038.4643 Km, e = 0.000890947,

i = 97.9411415 Deg, ω = 55.67025 Deg,

Ω = 182.00043 Deg, M = 87.03243 Deg.

The following table represents the comparison between our results and the published one on the NET.

The element	Our results	The published one	The difference
Time	2011/04/20 06:56:45.344	2011/04/20 03:01:53.344	3h 55m
a (Km)	7038.7	7038.800	0.1000 Km
e	0.000890947	0.0004666	0.00120
i (Deg.)	097.9411415	097.94230	0.1636000 Deg.
ω (Deg.)	055.6702500	269.23800	213.56775 Deg.
Ω (Deg.)	182.0004300	181.83680	0.1636000 Deg.

Notice that the argument of perigee (ω) is large difference that is because it depends on time.

Now, we have achievement our first goal of our work in this paper. The other goal in this work we studied the effects of forces on the motion of an artificial earth's satellites which are

i) the earth's gravitational field up to the fourth zonal harmonic, and

ii) the drag force with air density model (GOST Model).

And now we propagate our TLE (the initial condition) for one week (100 revolutions) with the value of 60 seconds as the time step and yield the following figures. These figures (Fig. 5.1 up to 5.5) show the variation of the classical orbital elements with the time over 100 revolutions with approximation of $w_\oplus$ equals the west-to-east angular velocity of the Earth.

6. CONCLUSION

We notice from the above Figures that the difference between perturbed drag force and Earth's gravitational force is clear due to the effect of these forces. While, the effect on the elements inclination and longitude of ascending node are not change & not significant that was expected, because of the inclination was only affected by the solar radiation pressure; and the EGYPTSAT-1 is Sun-synchronous satellite, so the longitude of ascending node is not affected.

Also, we can conclude that for seven mean solar days, there is obviously decay in the two elements (semi-major axis and eccentricity) but the other elements are lightly change except the inclination and longitude of ascending node. This expected because the only force affecting on the motion of artificial satellite is Earth's gravitational field and the drag force. These forces slightly affect on the elements (inclination, longitude of ascending node and argument

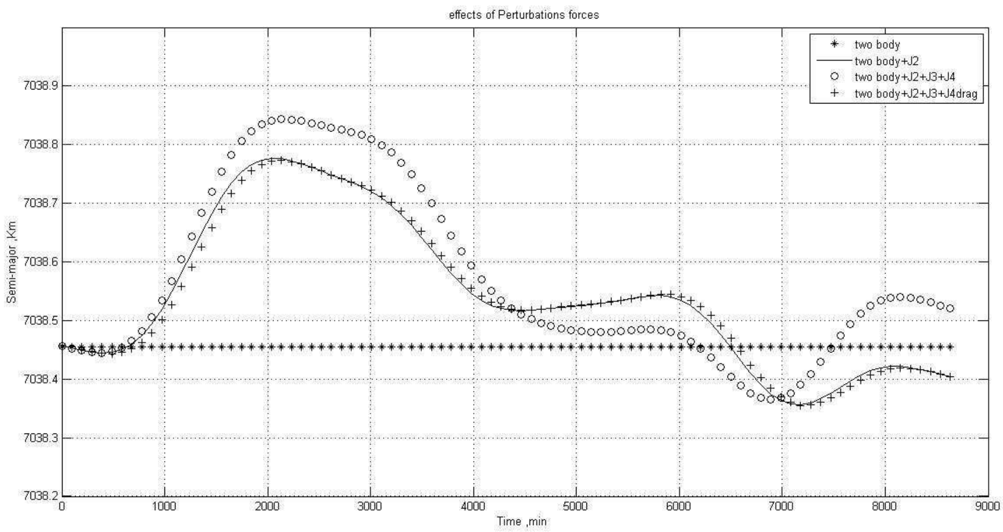

Fig.(5.1): Change of the semi-major axis under the perturbation force.

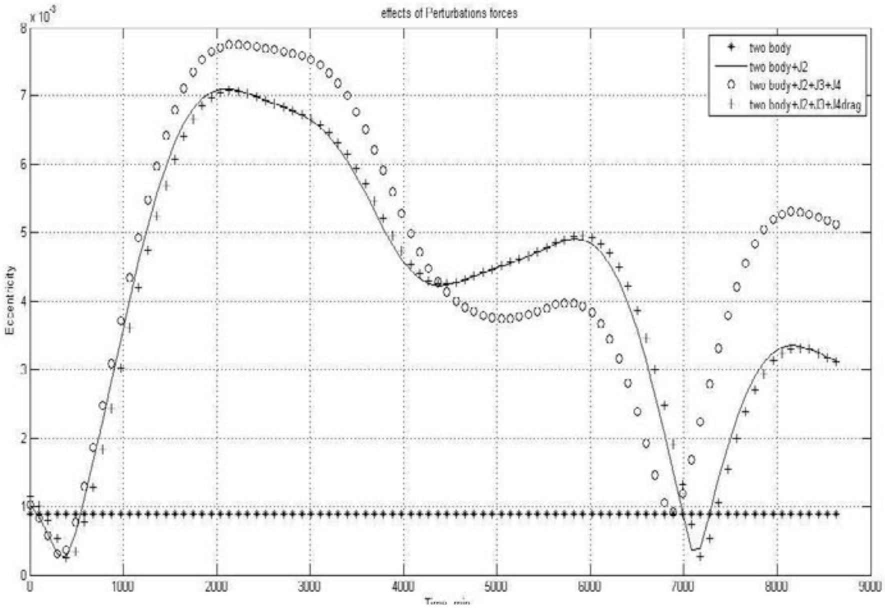

Fig.(5.2): Change of the eccentricity under the perturbation force.

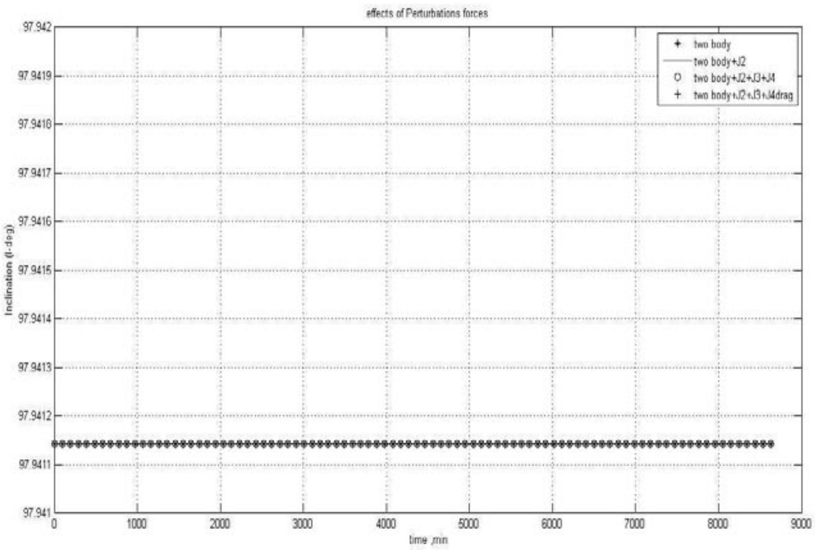

Fig.(5.3): Change of the inclination under the perturbation force.

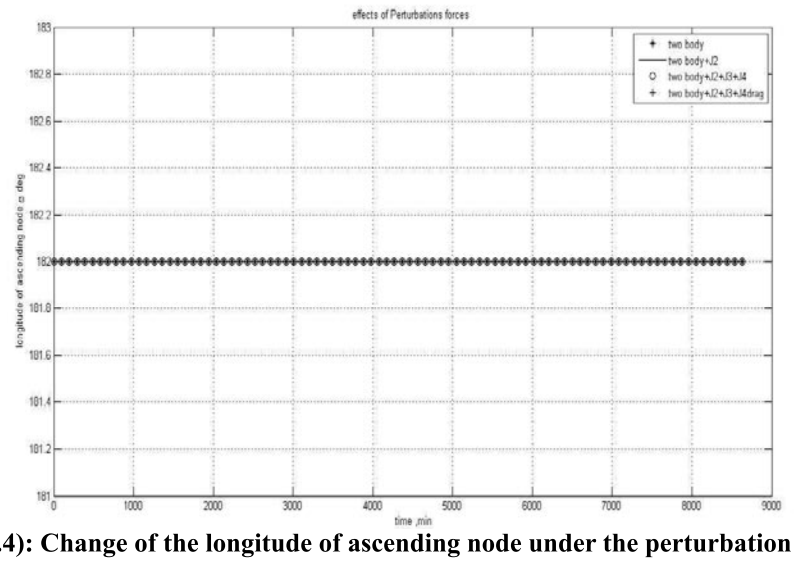

Fig.(5.4): Change of the longitude of ascending node under the perturbation force.

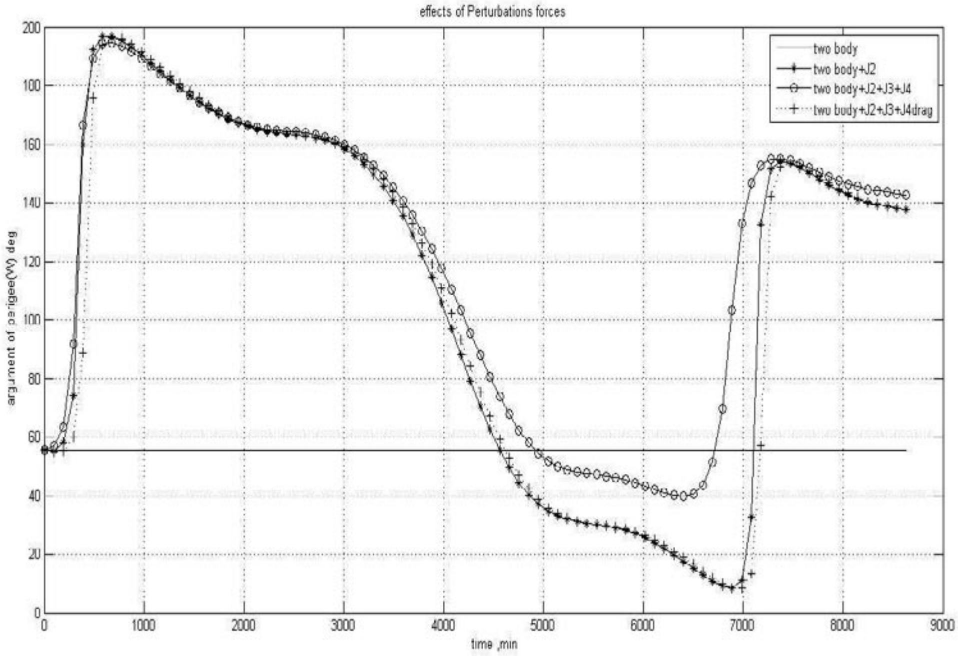

Fig.(5.5): Change of the argument of perigee under the perturbation force.

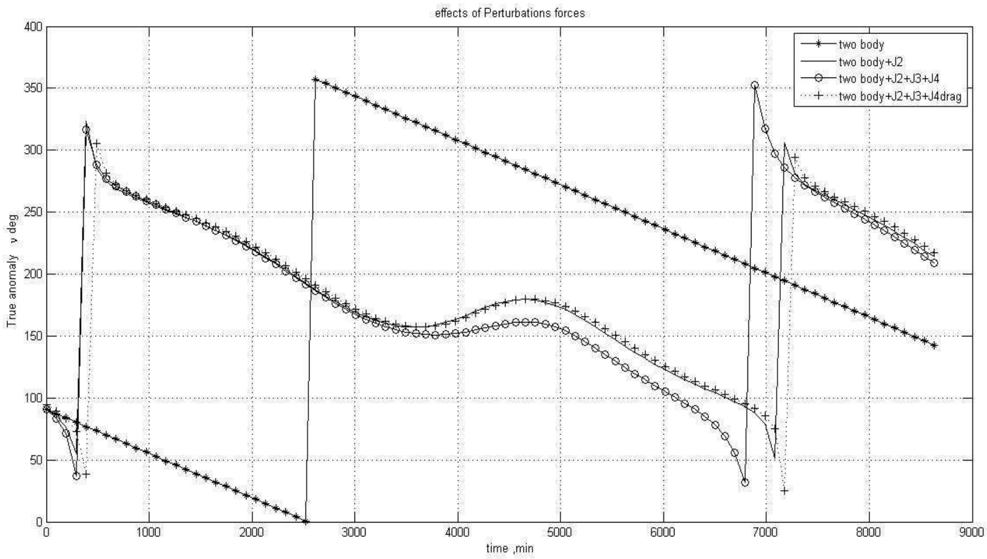

Fig.(5.6): Change of the true anomaly under the perturbation force.

of perigee) where these elements are strongly affected by the other forces like solar radiation pressure, and etc.

To get more accurate prediction of the motion of the artificial satellite we will be taken into account the whole other forces affecting on the motion of the artificial satellite.

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Al Azhar Bulletin of Science Vol. (25) No.1, June, 21 - 24, 2014,.

PERTURBATION EFFECT ON GROUND TRACKS OF SATELLITES ORBITS

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ABSTRACT

Effect of perturbations due to gravitational potential and drag on ground track of satellites orbits are studded. Components of velocity and position are obtained and the new orbital elements under the effect of perturbations are calculated to determine the latitudes and longitudes of the ground tracks.

1. INTRODUCTION

There are several sources of perturbations affecting satellite orbital motion from injection point until the end of its lifetime. In general orbit perturbations can be divided into gravitational and non-gravitational forces. The gravitational are those due to oblateness of the Earth and sectorial spherical harmonics and effect of sun/moon attraction. The non-gravitational perturbations include atmospheric drag force (the dominant for low earth orbits), solar radiation pressure (effective for geosynchronous satellites), magnetic forces (due to the interaction of the earth magnetic field with the dipole moment induced in the satellite), etc. The gravitational potential of the nonspherical earth models was initiated by (Kozai, 1959), short period and long period perturbations.

2. Equation of Motion with Perturbation

Knowledge of orbital motion is essential for a full understanding of space operations. Motion through space can be visualized using the laws described by Johannes Kepler and understood using the laws described by Sir Isaac Newton.

A satellite, under the influence of a perfect inverse square force field law, would have a set of constant orbital elements (a, e, i, M, Ω, ω). The general form of the equation of motion in a relative inertial coordinate system is given by

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{F} \qquad (2.1)$$

where $\vec{r}$ is the position vector of the satellite, μ is gravitational constant and $\vec{F}$ is the resultant vector of all the perturbing. $\vec{F}$ may consist of the following types of perturbation forces:

- Gravitational potential,
- Atmospheric drag.

In the presence of perturbations, the Keplerian orbit elements are no longer constant. The concept of variation of parameters allows the orbit elements to vary in such a way that, at any instant, the coordinates and velocity components can be computed from a unique set of two-body elements as if there were no perturbations. The equations of the variations can be derived from the concept of perturbed variations. There are two basic approaches to obtain the variation equations in celestial mechanics. They are the force components approach and the perturbing function approach.

The former is sometimes called the Gaussian method, and the latter is called the Lagrangian method (Rowa, 2002).

3. The Gauss Form of Lagrange's Equations

Now, summarize the formulae for the Gaussian form of the variation of parameter equations using the disturbing force with specific force components resolved in the *RSW* system (figure 1)

$$\frac{da}{dt} = \frac{2e\sin\theta}{n\chi}F_r + \frac{2a\chi}{nr}F_s, \qquad (3.1)$$

$$\frac{de}{dt} = \frac{\chi\sin\theta}{na}F_r + \frac{\chi}{na^2e}\left(\frac{a^2\chi^2}{r} - r\right)F_s, \qquad (3.2)$$

$$\frac{di}{dt} = \frac{r\cos u}{na^2\chi}F_w, \qquad (3.3)$$

$$\frac{d\Omega}{dt} = \frac{r\sin u}{na^2\chi\sin i}F_w, \qquad (3.4)$$

$$\frac{d\omega}{dt} = -\frac{\chi\cos\theta}{nae}F_r + \frac{p}{eh}\left[\sin\theta\left(1 + \frac{1}{1+e\cos\theta}\right)\right]F_s - \frac{r\cot i\sin u}{na^2\chi}F_w, \qquad (3.5)$$

$$\frac{dM}{dt} = n - \frac{1}{na}\left(\frac{2r}{a} - \frac{\chi^2}{e}\cos\theta\right)F_r - \frac{\chi^2}{nae}\left(1 + \frac{r}{a\chi^2}\right)\sin\theta F_s, \qquad (3.6)$$

where

- θ = true anomaly,
- n = mean motion,
- $\chi = \sqrt{1-e}$,
- $u = (\theta + \omega)$, ω argument of latitude,
- $p = a(1-e^2)$,
- $h = \sqrt{\mu p}$, and

F_r along the radius vector, F_s perpendicular to F_r in the orbit plane along motion and F_w normal to the orbit plane, such that the positive direction of (F_r, F_s, F_w) from a right-hand set of axes (Chobotov). If disturbing function $R = R(r, u, i)$, the components of disturbing force are given by

$$F_r = \frac{\partial R}{\partial r}, \quad F_s = \frac{1}{r}\frac{\partial R}{\partial u},$$

$$F_w = \frac{1}{r\,\sin(u)}\frac{\partial R}{\partial i}.$$

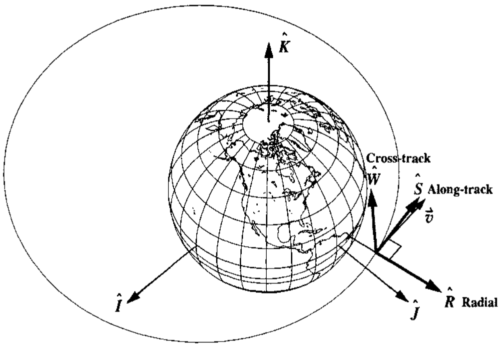

Fig.(1): Satellite Coordinate System in RSW.

This system moves with the satellite. The R-axis points to the satellite, the W-axis is normal to the orbital plane (and usually not aligned with the K-axis), and the W-axis is normal to the position vector. The W-axis is continuously aligned with the velocity vector only for circular orbits.

4. Perturbation Induced by Zonal Harmonic of the Geopotential

The potential function of the earth can be accurately expressed as an infinite series of zonal har

$$V = \frac{\mu}{r}\left[1 - \sum_{k=2}^{\infty} J_k \left(\frac{R_e}{r}\right)^k P_k(\sin L)\right], \qquad (4.1)$$

Where $P_k(\sin L)$ is the Legendre Polynomial of order k and L is the instantaneous latitude. The secular variation of the elements can be obtained by double average of disturbing function so $\left(\frac{da}{dt} = \frac{de}{dt} = \frac{di}{dt} = 0\right)$, (George, 1963).

5. Perturbation due to Drag Force

Drag is more important with lower orbits, where the atmosphere is more density that's miens more collision with satellite body. The atmospheric drag is expressed by the drag force per unit of mass in the following form (Frank, A. Marcos)

$$\vec{F}_{drag} = -\frac{1}{2} C_D \rho\, v^2 A. \qquad (5.1)$$

Divide both sides of equation by mass of satellite to obtain the acceleration of the atmospheric drag

$$\vec{a}_{drag} = -\frac{1}{2}\frac{C_D A}{m}\rho\, v^2 \frac{\vec{v}}{|\vec{v}|}, \qquad (5.2)$$

where A effective cross-section area, C_d drag coefficient and m is the satellite mass assuming a circular, equatorial orbit with an atmosphere rotates with the Earth, the satellite velocity vector with respect to the atmosphere, v, is defined as

$$\vec{v} = \vec{v}_{in} - \vec{w}_E \times \vec{r}, \qquad (5.3)$$

where $\vec{v}_{in}$ is the inertial velocity of the satellite $\vec{w}_E$ is the rotational velocity of the Earth, and $\vec{r}$ is the inertial satellite position vector. The drag coefficient, presented area, and mass may not be separately determinable, so these three quantities are usually grouped into a single quantity called the ballistic coefficient, B^*, which is defined as

$$B^* = \frac{C_D A}{m}.$$

From this definition, it can be seen that increasing the ballistic coefficient increases the amount of drag that acting on the satellite. Since the drag coefficient is relatively fixed, the ballistic coefficient can change only if the presented area of the satellite or the satellite mass is changed. From equations (3, 4.1 and 5.1) we can approximate changes in osculating orbital elements (George, 1963).

The main parameter affect the drag force is the density. The density of the upper atmosphere is expressed as exponential function of altitude given by

, $$\rho = \rho_0 \exp\left[-\frac{h_{ellp} - h_0}{H}\right] \quad (5.4)$$

where a reference density ρ_0, is used with the reference altitude, h_0 is the actual altitude h_{ellp} and the scale height, H are illustrated in Table (1) (Vallado, 2004).

Table (1): Atmospheric scale height & density.

Altitude (km)	Scale Height (km)	Atmospheric Density	
		(Mean (kg/m3	(Max (kg/m3
0	008.4	1.225	1.225
200	037.5	$10^{-10} \times 2.41$	$10^{-10} \times 3.65$
400	058.2	$10^{-12} \times 2.62$	$10^{-11} \times 1.05$
600	074.8	$10^{-14} \times 9.89$	$10^{-13} \times 8.46$
800	151.0	$10^{-15} \times 6.95$	$10^{-14} \times 9.41$
1000	296.0	$10^{-15} \times 1.49$	$10^{-14} \times 1.43$

Now, the elements of the orbit under perturbations can be expressed as (Escobal, 1965)

$$\Omega = \Omega_0 + \dot{\Omega}\Delta t\,, \quad (5.5.1)$$

$$w = w_0 + \dot{w}\,\Delta t\,, \quad (5.5.2)$$

$$a = a_0 + \dot{a}\,\Delta t\,, \quad (5.5.3)$$

$$e = e_0 + \dot{e}\,\Delta t\,, \quad (5.5.4)$$

$$I = I_0 + \dot{I}\,\Delta t\,, \quad (5.5.5)$$

$$M = M_0 + \dot{M}\,\Delta t\,, \quad (5.5.6)$$

where the initial elements (a_0, e_0, I_0, M_0, Ω_0, ω_0) and their accelerations are the variation of elements at instant of time Δt.

6. Satellite Ground Track

A ground track is the projection of the satellite's orbit onto the surface of the Earth (or whatever body the satellite is orbiting). We can determine the latitude and longitude of satellite from the following equations

$$X = r(\cos\Omega\cos u - \sin\Omega\sin u\cos i), \quad (6.1)$$

$$Y = r(\sin\Omega\cos u + \cos\Omega\sin u\cos i), \quad (6.2)$$

$$Z = r\sin i\sin u\,, \quad (6.3)$$

Where $$r = \frac{a(1-e^2)}{1+e\cos u}.$$

$$U = \sqrt{X^2 + Y^2 + Z^2}\,, \quad (6.4)$$

$$\sin d = Z/U\,, \quad (6.5.1)$$

$$\sin\delta = Z/U\,, \quad (6.5.1)$$

$$\sin\alpha = \frac{Y}{\sqrt{X^2+Y^2}}\,, \quad (6.5.2)$$

And $$\cos\alpha = \frac{X}{\sqrt{X^2+Y^2}}\,, \quad (6.5.3)$$

Then $$\lambda = \alpha - G.\,Sidereal\ Time\,, \quad (6.6.1)$$

$$\varphi = \delta - \varphi'\,, \quad (6.6.2)$$

Where φ' calculated from

$$\varphi' = \tan^{-1}\left[\frac{\tan\delta}{(1-f)^2}\right],$$

where f is the flatting of the earth.

7. Results and Conclusion

A computer program has been developed to solve the equation of orbital motion of two body problems with perturbations due to atmospheric drag force and the gravitational potential using Matlab. The variation of latitude &longitude of satellites was calculated. We applied these on the four satellites (YAOGAN 5, VANGUARD 3, USA 40 r and MOLNIYA 3-3) which TLE which obtains from *celectrack* web page as follow

YAOGAN 5

1 33456U 08064A 12159.19771302 .00012207 00000-0 34867-3 0 9002

2 33456 097.2574 230.6430 0011018 130.3255 314.5637 15.3609004319

VANGUARD 3

1 00020U 59007A 12158.38978192 .00000774 00000-0 30811-3 0 9586

2 00020 033.3463 172.0875 1683446 216.1252 131.3173 11.5189218389

USA 40 r

1 20344U 89061D 12156.94822004 0.00000190 00000-0 14430-3 0 09

2 20344 56.9980 113.9767 3572000 181.8041 178.1959 7.86216249 01

MOLNIYA 3-3

1 08425U 75105A 12158.75025176 -.00000305 00000-0 10000-3 0 1554

2 08425 063.7319 056.4327 7231782 244.5167 027.2896 02.0055820026

The results are shown in the following figures at revolution no. 500. Fig.(2) shows the effect of perturbation on the ground track of satellite YAOGAN 5. Fig.(3) shows the effect of perturbation on the ground track of satellite VANGUARD 3. Fig.(4) show the effect of perturbation on the ground track of satellite USA 40 r. Fig.(5) show the effect of perturbation on the ground track of satellite MOLNIYA 3-3.

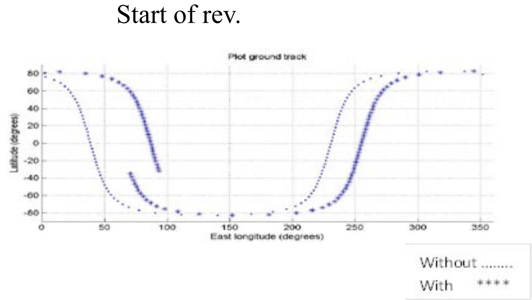

Fig.(2): Ground track of YAOGAN 5 satellite at rev. no. 500.

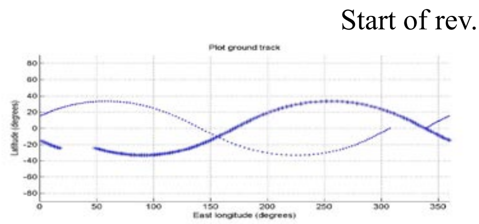

Fig.(3): Ground track of VANGUARD 3 satellite at rev. no. 500.

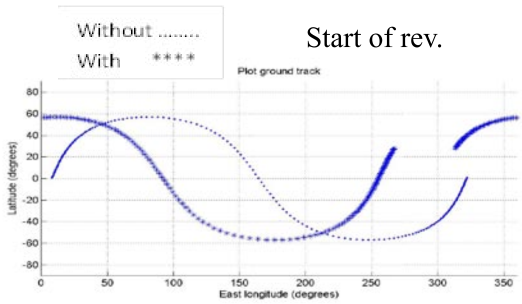

Fig.(4): Ground track of USA 40 r satellite at rev. no. 500.

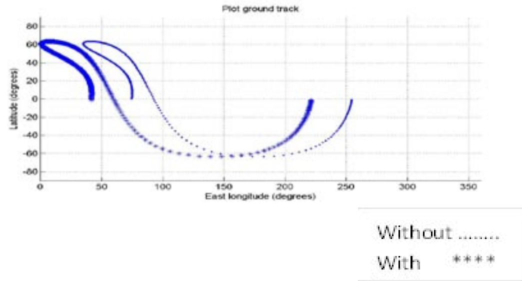

Fig.(5): Ground track of MOLNIYA 3-3 satellite at rev. no. 500.

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