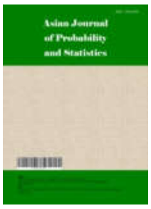

Asian Journal of Probability and Statistics

13(3): 44-61, 2021; Article no.AJPAS.69356
ISSN: 2582-0230

Generalized Rank Mapped Transmuted Distributions with Properties and Application: A Review

Imliyangba[1], Bhanita Das[1*] and Seema Chettri[1]

[1]Department of Statistics, North Eastern Hill University, Shillong, 793022, India.

Authors' contributions

This work was carried out in collaboration among all authors. All authors contributed immensely to the development of the article in all stages of the article formation. Author I designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BD managed the literature searches and author SC managed the analyses of the study. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2021/v13i330309
Editor(s):
(1) Dr. Manuel Alberto M. Ferreira, Lisbon University, Portugal.
Reviewers:
(1) Freddy Hernández Barajas, Universidad Nacional de Colombia, Colombia.
(2) Cardei Petru, The National Institute of Research, Romania.
Complete Peer review History: http://www.sdiarticle4.com/review-history/69356

Review Article

Received 24 March 2021
Accepted 04 June 2021
Published 10 June 2021

Abstract

Generalizing probability distributions is a very common practice in the theory of statistics. Researchers have proposed several generalized classes of distributions which are very flexible and convenient to study various statistical properties of the distribution and its ability to fit the real-life data. Several methods are available in the literature to generalize new family of distributions. The Quadratic Rank Transmutation Map (QRTM) is a tool for the construction of new families of non-Gaussian distributions and to modulate a given base distribution for modifying the moments like the skewness and kurtosis with the ability to explore its tail properties and improve the adequacy of the distribution. Recently, a new family of transmutation map, defined as Cubic Rank Transmutation (CRT) has been used by several authors to develop new distributions with application to real-life data. In this article, we have done a review work on the existing generalized rank mapped transmuted probability distributions, available in the literature with various statistical properties such as the reliability, hazard rate and cumulative hazard functions, moments, mean, variance, moment-generating function, order statistics, generalized entropy and quantile function along with its applications. Some future works have also been discussed for generalized rank mapped transmuted distributions.

**Corresponding author: Email: bhanitadas83@gmail.com;*

Keywords: Transmutation, Generalized Rank, Probability Distribution, Quadratic Transmutation, Cubic Rank Transmutation, Quartic Rank Transmutation.

1 Introduction

In many applied sciences such as medicine, engineering and finance, amongst others, modeling and analysing lifetime data are crucial. Several lifetime distributions have been used to model such kinds of data. The application of a statistical tool depends upon the underlying probability model of the data. As a result, huge numbers of probability distributions are being developed by numerous authors. However, there still remains large numbers of practical problems where the real data does not follow any of the classical or standard probability distributions. In the last two decades, several generalization approaches were adopted and practised which have received increased attention. Shaw and Buckley [1], developed an interesting method called the Quadratic Rank Transmutation Map (QRTM), which consists of introducing skewness or kurtosis in a symmetric or other (asymmetrical) distribution. Moreover, in order to capture the complexity of the data and increases the flexibility, new classes of Cubic Rank Transmutation (CRT) have been developed by Granzotto et al., [2]. Using this concept of CRT, various authors have proposed several cubic transmuted probability distributions which show better flexibility to handle more complex (bi-modal) data over quadratic transmuted distributions.

2 Developments in Transmuted Distributions

According to Shaw and Buckley [1], the cumulative distribution function (*cdf*) of the QRTM has the following simple quadratic form as

$$F(x) = (1+\lambda)G(x) - \lambda G(x)^2\,, \tag{1}$$

where $\epsilon\, \mathbb{R}$, $\lambda \in [-1, 1]$, $G(x)$ is the *cdf* of the baseline distribution.

2.1 Quadratic rank transmutation map

Aryal and Tsokos [3] first emphasized on the technique given in (1) and introduced Transmuted Extreme Value distributions that would provide more distributional flexibility in reliability analysis. Further, various authors have developed several probability distributions using the QRTM given in (1) for various choices of baseline *cdf* $G(x)$. Tahir and Cordeiro [4] and Rahman et al., [5] have provided a list for various quadratic transmuted distributions. At present, transmuted distributions are very common in the literature. An updated list of popular transmuted-G classes of distributions with its applications is given in Table 1.

Moreover, according to Granzotto et al., [2], the construction of the QRTM is simple and intuitive. Let X_1 and X_2 be independent and identically distributed random variables with distribution $G(x)$. Then, consider

$Y \stackrel{\text{def}}{=} min(X_1, X_2)$, with probability π,

$Y \stackrel{\text{def}}{=} max(X_1, X_2)$, with probability $1-\pi$,

where $0 \le \pi \le 1$. The distribution of Y is evidently

$$F_Y(x) = \pi \Pr(min(X_1, X_2) \le x) + (1-\pi)\Pr(max(X_1, X_2) \le x)$$

where Pr is the probability of an event.

We know that $F_{min}(x) = 1 - [1 - G(x)]^n$ and $F_{max}(x) = [G(x)]^n$

$$F_Y(x) = \pi[1-(1-G(x))^2] + (1-\pi)G^2(x) = 2\pi G(x) + (1-2\pi)G^2(x)$$

If we take $2\pi = \lambda$, the distribution is the well-known QRTM [2].

$$F(x) = \lambda G(x) + (1-\lambda)G(x)^2 \tag{2}$$

Observe that at $\lambda = 1$ in (2), the above distribution gives the baseline distribution.

Table 1. Development in quadratic rank transmuted distributions

Sl. No.	Author(s) (Year)	Distribution	Applications
1	Aryal and Tsokos [3]	Transmuted Extreme Value	Snowfall data in midway airport (Illinois, 1970-2007)
2	Aryal and Tsokos [6]	Transmuted Weibull	(i) Tensile fatigue charateristics of a polyster (ii) Breaking of stress of carbon fibres
3	Aryal [7]	Transmuted Log-logistic	Theoretical development without application
4	Merovci [8]	Transmuted Exponentiated Exponential	Strength of 1.5cm glass fibres (England)
5	Mahmoud and Mandouh [9]	Transmuted Fréchet	(i) Breaking of stress of carbon fibres (ii) Strength of glass fibres
6	Ashour and Eltehiwy [10]	Transmuted Lomax	Theoretical development without application
7	Merovci [11]	Transmuted Lindley	Sample of 128 bladder cancer patients
8	Elbatal and Elgarhy [12]	Transmuted Quasi-Lindley	Theoretical development without application
9	Ashour and Eltehiwy [13]	Transmuted Exponentiated-Lomax	Theoretical development without application
10	Elbatal [14]	Transmuted Generalized Inverted Exponential	Theoretical development without application
11	Elbatal [15]	Transmuted Modified Inverse Weibull	Theoretical development without application
12	Ashour and Eltehiwy [26]	Transmuted Exponentiated Modified Weibull	Theoretical development without application
13	Elbatal et al., [17]	Transmuted Generalized Linear Exponential	(i) Lifetimes of 40 patients suffering from leukemia (Saudi Arabia) (ii) Lifetimes of 50 devices (Aarset,1987)
14	Merovci et al., [18]	Transmuted Generalized Inverse Weibull	Failure times of 50 items
15	Merovci and Elbatal [19]	Transmuted Lindley-Geometric	Waiting times before services of 100 bank customers
16	Elbatal and Aryal [20]	Transmuted Additive Weibull	Lifetimes of 50 devices (Aarset,1987)
17	Khan and King [21]	Transmuted Modified Weibull	Maximum flood level (Pennsylvania)
18	Merovci and Puka [22]	Transmuted Pareto	Exceedances of flood peaks of the Wheaton River (Canada)

Sl. No.	Author(s) (Year)	Distribution	Applications
19	Iriarte and Astorga [23]	Transmuted Maxwell	Energy consumption during a certain period for a sample of 90 homes
20	Tian et al., [24]	Transmuted Linear Exponential	Lifetimes of 50 devices (Aarset,1987)
21	Ahmad et al., [25]	Transmuted Inverse Rayleigh	Theoretical development without application
22	Merovci [26]	Transmuted Generalized Rayleigh	Nicotine measurements made in several brands of cigarettes in 1998
23	Khan and King [27]	Transmuted Inverse Weibull	Survival times of 128 bladder cancer patients
24	Merovci et al., [28]	Transmuted Genereralized Inverse Weibull	Theoretical development without application
25	Khan and King [29]	Transmuted Generalized Inverse Weibull	Theoretical development without application
26	Hussian [30]	Transmuted Exponentiated Gamma	Theoretical development without application
27	Elbatal et al., [31]	Transmuted Exponentiated Fréchet	Windspeed (Carmeron Highland, Malaysia)
28	Ahmad et al., [32]	Transmuted Weibull	Theoretical development without application
29	Lucena et al., [33]	Transmuted Generalized Gamma	Secondary Reactor Pumps
30	Owoloko et al., [34]	Transmuted Exponential	Theoretical development without application
31	Abdul-Moniem [35]	Transmuted Burr Type III	Life of fatigue fracture of Kevlar 373/epoxy
32	Abdul-Moniem and Seham [36]	Transmuted Gompertz	Life of fatigue fracture of Kevlar 373/epoxy
33	Khan and King [37]	Transmuted Modified Inverse Rayleigh	30 successive values of March Precipitation (in inches)
34	Mansour et al., [38]	Transmuted Additive Weibull	(i) Ages for 155 patients of breast tumors (Egypt) (ii) Failure time of 50 items reported in Aarset (1987)
35	Afify et al., [39]	Transmuted Marshall-Olkin Frechet	(i) Breaking stress of carbon fibres (ii) Strengths of 1.5cm glass fibres (England)
36	Afify et al., [40]	Transmuted Weibull-Lomax	Gauge lengths of 10mm
37	Iriarte and Astorga [41]	Transmuted Generalized Rayleigh	Break life by fatigue of Kevlar 49/epoxy filaments
38	Granzotto and Louzada [42]	Transmuted Log-Logistic	Tabapua Race Cow data (Brazil)

Sl. No.	Author(s) (Year)	Distribution	Applications
39	Elbatal and Aryal [43]	Transmuted Dagum	A fleet of 13 Boeing 720 jet airplanes (Proschan, 1963)
40	Luguterah and Nariru [44]	Transmuted Exponential Pareto	Fatigue life of 6061-T6 Aluminium coupons Data
41	Fatima and Roohi [45]	Transmuted Exponentiated-Pareto	(i) Breaking stress of carbon fibres (ii) Strengths of 1.5cm glass fibres (England)
42	Mansour and Mohamed [46]	Transmuted Lindley	128 bladder cancer patients
43	Khan et al., [47]	Transmuted Chen Lifetime	Strengths glass of fibres data.
44	Khan et al., [48]	Transmuted Kumaraswamy	(i) Flood data (ii) Infants born to HIV+ve women
45	Elgarhy et al., [49]	Transmuted Generalized Lindley	Theoretical development without application
46	Vardhan and Balaswamy [50]	Transmuted Modified Weibull	Theoretical development without application
47	Afify et al., [51]	Transmuted Weibull-Pareto	Gauge lengths of 10 mm
48	Shahzad and Asghar [52]	Transmuted Dagum	Rainfall data for the city of Islamabad, Pakistan
49	Khan et al., [53]	Transmuted Gompertz	Failure times of windshields data
50	Haq et al., [54]	Transmuted Power Function	(i) Strengths of 1.5 cm glass fibres (ii) Failure times of 50 items
51	Chakraborty and Bhati [55]	Transmuted Geometric	(i) A fleet of 13 Boeing 720 jet airplanes (ii) Vinyl chloride concentration (iii) Protein amount for adult patients (Chilean Hospital)
52	Bourguignon et al., [56]	Transmuted Birnbaum-Saunders	(i) Bladder cancer data (ii) ICU data
53	Khan et al., [57]	Transmuted Generalized Exponentiated Exponential	(i) Carbon fibre data (ii) Bladder cancer data
54	Cordeiro et al., [58]	Transmuted Modified Weibull	(i) Survival times of 72 guinea pigs infected with virulent tubercle bacilli
55	Elgarhy et al., [59]	Transmuted Generalized Quasi Lindley	(ii) March precipitation (in inches) in Minneapolis/St Paul
56	Khan et al., [60]	Transmuted Generalized Inverse Weibull	(i) Ball bearings data (ii) Fatigue life of aluminium data
57	Khan et al., [61]	Transmuted Weibull	(i) Nicotine measurements made in several brands of cigarettes in 1995 (ii) Headache relief patient's

Sl. No.	Author(s) (Year)	Distribution	Applications
			data
58	Al-Babtain et al., [62]	Transmuted Kumaraswamy Exponentiated Modified Weibull	(i) Nicotine measurements made from several brands of cigarettes in 1998 (ii) Greenwich data
59	Chhetri et al., [63]	Transmuted Kumaraswamy Pareto	(i) Exceedances of flood peaks of the Wheaton River (ii) Norwegian fire insurance data
60	Deka et al., [64]	Transmuted Exponentiated Gumbel	Water quality data using some water quality parameters
61	Al-Omari et al., [65]	Transmuted Janardan	Theoretical development without application
62	Jayakumar et al., [66]	Transmuted T-X Family of Distributions	(i) Life of fatigue of Kelvar 373/epoxy (ii) Survival times of 121 patients with breast cancer
63	Venegas et al., [67]	Transmuted Exponentiated Maxwell	Single edge V-notched Aluminium plate repaired with Kevlar 49/epoxy
64	Nofal et al., [68]	Transmuted Exponentiated Additive Weibull	(i) Breaking stress of carbon fibres (ii) Nicotine measurements made in several brands of cigarettes
65	Pobočíková et al., [69]	Transmuted Weibull	(i) Lifetimes of Kevlar 49/ epoxy strands. (ii) Survival times of 72 guinea pigs (iii) 155 patients suffering from breast cancer
66	Arshad et al., [70]	Transmuted Exponentiated Moment Pareto	(i) Exceedances of flood peaks of the Wheaton River (ii) Remission times of bladder cancer 128 patients (iii) Kevlar 49/epoxy strands failure times (iv) Waiting time before the customer receives service in a bank
67	Khan et al., [71]	Transmuted Modified Weibull	Nicotine measurements made in several brands of cigarettes in 1995
68	Abdullahi and Ieren [72]	Transmuted Exponential Lomax	Theoretical development without application
69	Elgarhy et al., [73]	Transmuted Kumaraswamy Quasi Lindley	(i) Strength data of glass of the aircraft window (ii) Relief times of 20 patients receiving an analgesic (iii) Waiting times before services of 100 bank customers
70	Balaswamy [74]	Transmuted Half Normal	(i) March precipitation (in inches) in Minneapolis/St Paul

Sl. No.	Author(s) (Year)	Distribution	Applications
			(ii) Survival times of 72 guinea pigs infected with virulent tubercle bacilli
71	Haq et al., [75]	Transmuted Weibull Power Function	(i) Strengths of 1.5 cm glass fibres (ii) Breaking stress of carbon fibres
72	Tahir *et at.*, [76]	Transmuted New Weibull-Pareto	(i) Exceedances of flood peaks of the Wheaton River (ii) Floyd River flood rates for years 1935–1973
73	Khan [77]	Transmuted Generalized Inverted Exponential	Survival times for the 50 devices
74	Afify et al., [78]	Transmuted Burr XII	(i) The Gauge Lengths Data (ii) The Nicotine Data (iii) Vitamin A data regression model
75	Okorie and Akpanta [79]	Transmuted Generalized Inverted Expo.	50 devices put on life test at time zero
76	Abayomi [80]	Transmuted Half Normal	Buying behaviour data culled from a standard wholesale outlet
77	Khan et al., [81]	Transmuted Burr Type X	Fatigue fracture data and multiple myeloma patient's data.
78	Gharaibeh and Al-Omari [82]	Transmuted Ishita	Fatigue fracture of kevlar 373/epoxy
79	Otiniano et al., [83]	Transmuted Generalized Extreme Value	Stock market indices: Ibovespa, S&P 500 and Dow Jones
80	Samuel [84]	Transmuted Logistic	March precipitation in Minneapolis/ St Paul.
81	Khan [85]	Transmuted Modified Inverse Weibull	Survival remission times of bladder cancer data
82	Khan et al., [86]	Transmuted Exponentiated Weibull	Failure times of 50 components (per 1000h) data
83	Ishaq et al., [87]	Transmuted Generalized Rayleigh	(i) Flood data sets with 20 observations (ii) Time of failure and running times for a sample of devices
84	Riffi et al., [88]	Generalized Transmuted Fréchet	Leukemia free-survival times for the 46 autologous transplant patients
85	Menezes et al., [89]	Transmuted Half-Normal	Daily series of daily precipitation
86	Aijaz et al., [90]	Transmuted Inverse Lindley	(i) Relief times of 20 patients getting an analgesic (ii) Breaking stress of carbon fibres
87	Yadav et al., [91]	Transmuted Lifetime	128 bladder cancer patients

Sl. No.	Author(s) (Year)	Distribution	Applications
88	Sarabia et al., [92]	Transmuted Bivariate Distributions	(i) Reliability analysis of cable insulation specimens (ii) Reliability analysis of two-component parallel systems (iii) Sports data
89	Badr et al., [93]	Transmuted Odd Fréchet-G Family of Distributions	(i) Reliability analysis (ii) 72 measurements of exceedances of the Wheaton River in Canada (iii) Vehicle insurance losses

2.2 Cubic rank transmutation map

Recently, a new family of transmutation map, named Cubic Rank Transmutation (CRT) is introduced by Granzotto et al., [2]. They developed the CRT log-logistic and CRT Weibull distributions which offer tractable distributions and are able to fit complex data sets such as ones with bimodal distribution or bimodal hazard rates.

Let X_1, X_2 and X_3 be independent and identically random variables distributed with distribution $G(x)$. Now consider the following order.

$$X_{1:3} = \min\,(X_1, X_2, X_3),\ X_{2:3} = \text{the } 2^{\text{nd}} \text{ smallest of } (X_1, X_2, X_3) \text{ and } X_{3:3} = \max\,(X_1, X_2, X_3)$$

And let $Y \overset{\text{def}}{=} X_{1:3}$, with probability π_1,

$Y \overset{\text{def}}{=} X_{2:3}$, with probability π_2,

$Y \overset{\text{def}}{=} X_{3:3}$, with probability π_3,

Where $\sum_{i=1}^{3} \pi_i = 1 \Longrightarrow \pi_3 = 1 - \pi_1 - \pi_2$. Evidently $F_Y(x)$ is given by

$$\begin{aligned} F_Y(x) &= \pi_1 \Pr(min(X_1, X_2, X_3) \le x) + \pi_2(\Pr X_{2:3} \le x) + \pi_3 \Pr(min(X_1, X_2, X_3) \le x) \\ &= 3\pi_1 G^2(x) + 3(\pi_2 - \pi_1)G^2(x) + (1 - 3\pi_2)G^3(x) \end{aligned}$$

And if $3\pi_1 = \lambda_1$ and $3\pi_2 = \lambda_2$, the above distribution becomes

$$F(x) = \lambda_1 G(x) + (\lambda_2 - \lambda_1)[G(x)]^2 + (1 - \lambda_2)[G(x)]^3 \tag{3}$$

We see that at $\lambda_1 = \lambda_2 = 1$, the above distribution gives the baseline distribution.

Moreover, Rahman et al., [94-96] have introduced three new cubic transmuted families of distributions which are defined as follows in equation (4), (5) and (6) respectively.

$$F\,(x) = (1 + \lambda_1)G(x) + (\lambda_2 - \lambda_1)G^2(x) - \lambda_2 G^3(x), x \in R, \tag{4}$$

where $\lambda_1 \in [-1, 1]$ and $\lambda_2 \in [-1, 1]$ and $-2 \le \lambda_1 + \lambda_2 \le 1$. It can be easily observed that the cubic transmuted family of distributions proposed by AL-Kadim and Mohammed [97] turned out to be a special case of (4) for $\lambda_2 = -\lambda_1$.

$$F\,(x) = (1 + \lambda_1 + \lambda_2)G(x) - (\lambda_1 + 2\lambda_2)G^2(x) + \lambda_2 G^3(x), x \in R, \tag{5}$$

where $\lambda_1 \in [-1, 1]$ and $\lambda_2 \in [0, 1]$.

$$F(x) = (1-\lambda)G(x) + 3\lambda G^2(x) - 2\lambda G^3(x),\ x \in R, \quad (6)$$

where $\lambda \in [-1, 1]$.

Aslam *et at.*, [98], introduced another cubic transmuted-G family of distributions and its related statistical properties. The lists of several cubic transmuted distributions introduced by various researchers are mentioned in Table 2.

Table 2. Development in cubic rank transmuted distributions

Sl.No.	Authors (Year)	Distribution	Applications
1	Granzotto et al., [2]	Cubic Transmuted Weibull	Breaking stress of carbon fibres
2	Granzotto et al., [2]	Cubic Transmuted Log-Logistic	Cattle sexual precocity data
3	AL-Kadim and Mohammed [97]	Cubic Transmuted Weibull	Theoretical development without application
4	Rahman et al., [5]	Cubic Transmuted Exponential	(i) Lifetimes of 50 devices (Aarset,1987) (ii) Electronics Data
5	Rahman et al., [99]	Cubic Transmuted Pareto	(i) Life of fatigue fracture of Kevlar 373/epoxy (ii) Floyd River Dataset
6	Ansari and Eledum [100]	Cubic Transmuted Pareto	(i) Wheaton River Flood Peaks Data Set (ii) Floyd River Flood Data Set
7	Rahman et al., [95]	Cubic Transmuted Exponential	(i) The Wheaton River Data (ii) The Floyd River Data
8	Celik [101]	Cubic Transmuted Fréchet	Wind speed data
9	Celik [101]	Cubic Transmuted Gumbel	Water Quality Data
10	Celik [101]	Cubic Transmuted Gompertz	Failure Data
11	Saraçoğlu and Tanış [102]	Cubic Rank Transmuted Kumaraswamy	(i) Milk production data (ii) Operation and empirical data
12	Riffi and Hamdan [103]	Cubic Transmuted Gompertz-Makeham	Theoretical development without application
13	Ansari et al., [104]	Cubic Transmuted Power Function (CTPFD)	(i) 100 data points simulated from CTPFD (ii) 72-hour acute salinity tolerance of river marine invertebrates (iii) Failure times of 50 components
14	Rahman et al., [105]	Cubic Transmuted Weibull	(i) Carbon Fibres Data (ii) The Wheaton River Data
15	Bhatti et al., [106]	Cubic Transmuted Burr III-Pareto	(i) Tensile strength of carbon fibres (ii) Strengths of glass fibres
16	Rahman et al., [96]	Cubic Transmuted Uniform	Lifetimes of 30 electronic devices
17	Adeyinka [107]	Cubic Transmuted Exponentiated Exponential	Infant mortality rate per 1,000 live births in Nigeria

Sl.No.	Authors (Year)	Distribution	Applications
18	Eledum [108]	Cubic Transmuted Exponentiated Pareto-1	(i) Failure times of Kevlar 49/epoxy strands (ii) Failure Times (in hours) of 50 Components
19	Ogunde et al., [109]	Cubic Transmuted Gompertz	Remission times of a random sample of 128 bladder cancer patients
20	Akter et al., [110]	Cubic Transmuted Burr-XII	(i) Life of fatigue fracture of Kevlar 373/ epoxy (ii) Remission times of a random sample of 128 bladder cancer patients
21	Ali et al., [111]	Cubic Transmuted Weibull	Theoretical development without application

2.3 Quartic Rank Transmutation Map

We can easily obtain the Quartic Transmuted Families of Distributions using the concept of Granzotto et al., [2]. Let X_1, X_2, X_3 and X_4 be independent and identically random variables distributed with distribution $G(x)$. Now consider the following order.

$$X_{1:4} = \min\,(X_1 X_2, X_3, X_4, X_{2:4} \text{ the 2nd smallest of } (X_1, X_2, X_3, X_4)\,,$$
$$X_{3:4} = \text{the 3rd smallest of } (X_1, X_2, X_3, X_4) \text{ and } X_{4:4} = \max\,(X_1, X_2, X_3, X_4),$$

And let

$$Y \overset{\text{def}}{=} X_{1:4}, \quad \text{with probability } \pi_1,$$

$$Y \overset{\text{def}}{=} X_{2:4}, \quad \text{with probability } \pi_2,$$

$$Y \overset{\text{def}}{=} X_{3:4}, \quad \text{with probability } \pi_3,$$

$$Y \overset{\text{def}}{=} X_{4:4}, \quad \text{with probability } \pi_4,$$

Where $\sum_{i=1}^{4} \pi_i = 1 \Rightarrow \pi_4 = 1 - \pi_1 - \pi_2 - \pi_3$. Evidently $F_Y(x)$ is given by

$$F_Y(x) = \pi_1 \Pr(\min(X_1, X_2, X_3, X_4) \le x) + \pi_2(\Pr(X_{2:4}) \le x) + \pi_3(\Pr(X_{3:4}) \le x) + (1 - \pi_1 - \pi_2 - \pi_3) \Pr\,(\max(X_1, X_2, X_3, X_4) \le x)$$
$$= 2(2\pi_1)G(x) + 3(2\pi_2 - 2\pi_1)G^2(x) + 2(2\pi_1 - 4\pi_2 + 2\pi_3)G^3(x) + (1 - 2\pi_1 + 2\pi_2 - 4\pi_3)G^4(x)$$

And if $2\pi_1 = \lambda_1$ and $2\pi_2 = \lambda_2$, $2\pi_3 = \lambda_3$, the above distribution becomes

$$F(x) = 2\lambda_1 G(x) + 3(\lambda_2 - \lambda_1)G^2(x) + 2(\lambda_1 - 2\lambda_2 + \lambda_3)G^3(x) + (1 - \lambda_1 + \lambda_2 - 2\lambda_3)G^4(x) \qquad (7)$$

At $\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{2}$ in (7), the above distribution becomes the baseline distribution.

Utilizing the above equation (7) there is a huge scope to develop quartic rank transmuted probability distributions. Moreover, using the same concept we can generalize the n^{th} rank mapped transmuted distributions.

Ali et al., [111] generated a new Generalized Rank Mapped Transmuted Distribution for generating families of continuous distributions which is defined as follows

Generalized transmuted *cdf* of n^{th} rank mapped (n = 1, 2, 3, …) distribution is given by

$$F_Y(x) = \sum_{r=1}^{n} \pi_{r:n} I_{G(x)}(r, n-r+1)$$

$$= \sum_{r=1}^{n} m_{r:n}(x) \tag{8}$$

where, $I_{G(x)}(r, n-r+1)$ is incomplete beta function ratio and the corresponding generalized transmuted probability distribution function(*pdf*) of n^{th} rank mapped distribution is

$$f_Y(x) = g(x)[G(x)]^{n-1} \sum_{r=1}^{n} \sum_{j=0}^{n-r} k_{rj}(x)$$

$$= g(x)[G(x)]^{n-1} \sum_{r=1}^{n} k_r \tag{9}$$

where $g(x)$ is the *pdf* of a continuous population drawn from a random sample of size n,

$$k_i = \sum_{j=0}^{n-i} k_{ij}$$

and

$$k_{ij} = \frac{(-1)^{n-i-j} \pi_{i:n}}{B(i,\ n-i+1)} \binom{n-i}{j} [G(x)]^{-j}$$

3 Conclusion

Generalization of probability distributions through transmutation was first applied in the area of financial mathematics. As a result, several researchers have successfully applied this technique to model lifetime and survival data. At present, this approach is being applied in the areas of biology, engineering, environmental, medical among others to handle more complex (bi-modal) data.

In this work, we have reviewed the rank transmuted distributions which includes the quadratic and cubic rank transmuted distributions. We have also provided the quadratic and cubic rank transmuted distributions in table 1 and 2, along with their respective authors and applications. We expect that this review work will be of great value in the field of statistics.

As for the scope of future, Bivariate and multivariate transmuted rank distributions along with its properties can be studied. Also, Bayesian statistical inference which is one of the most important areas of research can be done for generalized rank mapped Transmuted distribution.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Shaw WT, Buckley IRC. The alchemy of probability distributions: Beyond gram-charlier expansions, and a skew-kurtotic-normal distribution from a rank transmutation map. Research Report; 2007.

[2] Granzotto DCT, Louzada F, Balakrishnan N. Cubic rank transmuted distributions: Inferential issues and applications. Journal of Statistical Computation and Simulation. 2017;87:2760–2778.
DOI: 10.1080/00949655.2017.1344239.

[3] Aryal GR, Tsokos CP. On the transmuted extreme value distribution with application. Non- linear Analysis: Theory, Methods and Applications. 2009;71:1401-1407.
DOI: 10.1016/j.na.2009.01.168.

[4] Tahir MH, Cordeiro GM. Compounding of distributions: A survey and new generalized classes. Journal of Statistical Distributions and Applications. 2016;3:1–35.
DOI: 10.1186/s40488–016–0052–1.

[5] Rahman MM, Al-Zahrani B, Shahbaz MQ. A general transmuted family of distributions. Pakistan Journal Statistical Operation and Research. 2018a;14:451–469.
DOI: 10.18187/pjsor.v14i2.2334.

[6] Aryal GR, Tsokos CP. Transmuted Weibull distribution: A generalization of the Weibull probability distribution. European Journal of Pure and Applied Mathematics. 2011;4:89-102.

[7] Aryal GR. Transmuted log-logistic distribution. Journal of Statistics Applications and Probability. 2013;2:11–20.

[8] Merovci F. Transmuted exponentiated exponential distribution. Mathematical Sciences and Applications E-Notes. 2013a;1:112–122.

[9] Mahmoud MR, Mandouh RM. On the transmuted Fréchet distribution. Journal of Applied Sciences Research. 2013;9:5553–5561.

[10] Ashour SK, Eltehiwy MA. Transmuted lomax distribution. American Journal of Applied Mathematics and Statistics. 2013c;1:121–127.
DOI: 10.12691/ajams–1–6–3.

[11] Merovci F. Transmuted lindley distribution. International Journal of Open Problems in Computer Science and Mathematics. 2013b;6:63–72.

[12] Elbatal I, Elgarhy M. Transmuted quasi-Lindley distribution: A generalization of the quasi- Lindley distribution. International Journal of Pure Applied Science and Technology. 2013;18:59–70.

[13] Ashour SK, Eltehiwy MA. Transmuted exponentiated lomax distribution. Australian Journal of Basic and Applied Sciences. 2013a;7:658–667.

[14] Elbatal I. Transmuted generalized inverted exponential distribution. Economic Quality Control. 2013a;28:125– 133.
DOI: 10.1515/eqc–2013–0020.

[15] Elbatal I. Transmuted modified inverse Weibull distribution: A generalization of the modified inverse Weibull probability Distribution. International Journal of Mathematical Archive. 2013b;4:117–119.

[16] Ashour SK, Eltehiwy MA. Transmuted exponentiated modified Weibull distribution. International Journal of Basic and Applied Sciences. 2013b;2:258–269.
DOI: 10.14419/ijbas.v2i3.1074.

[17] Elbatal I, Diab LS, Alim NAA. Transmuted generalized linear exponential distribution. International Journal of Computer Applications. 2013;83:29–37.

[18] Merovci F, Elbatal I, Ahmed A. Transmuted generalized inverse Weibull distribution. Australian Journal of Statistics. 2014;43:119–131.

[19] Merovci F, Elbatal I. Transmuted lindley-geometric distribution and its applications. Journal of Statistics Applications and Probability. 2013;1:77-91.

[20] Elbatal I, Aryal G. On the transmuted additive Weibull distribution. Australian Journal of Statistics. 2013; 42:117–132.
DOI: 10.17713/ajs.v42i2.160.

[21] Khan M, King R. Transmuted modified Weibull distribution: A generalization of the modified Weibull probability distribution. European Journal of Pure and Applied Mathematics. 2013;6:66–88.

[22] Merovci F, Puka L. Transmuted Pareto Distribution. Prob Stat Forum. 2014;7:1–11.

[23] Iriarte YA, Astorga JM. Transmuted maxwell probability distribution. Revista Integracio´n. 2014;32:211 221.

[24] Tian Y, Tian M, Zhu Q. Transmuted linear exponential distribution: A new generalization of the linear exponential distribution. Communications in Statistics-Simulation and Computation. 2014;43:2661–2671.
DOI: 10.1080 /03 610 918.2013.763978.

[25] Ahmad A, Ahmad SP, Ahmad A. Transmuted inverse rayleigh distribution: A generalization of the inverse Rayleigh distribution. Mathematical Theory and Modeling. 2014;4:90–98.

[26] Merovci F. Transmuted generalized Rayleigh distribution. Journal of Statistics Applications and Probability. 2014;3:9–20.
DOI: 10.18576/jsap/030102.

[27] Khan MS, King R. A new class of transmuted inverse Weibull distribution for reliability analysis. American Journal of Mathematical and Management Sciences. 2014a;33:261–286.
DOI: 10.1080/ 01966324. 2014.929989.

[28] Merovci F, Alizadeh M, Hamedani GG. Another generalized transmuted family of distributions: Properties and applications. Austrian Journal of Statistics. 2016;45:71–93.
DOI: 10.17713/ajs.v45i3.109.

[29] Khan MS, King R. Transmuted generalized inverse Weibull distribution. Journal of Applied Statistical Science. 2014b;20:213–230.

[30] Hussian MA. Transmuted exponentiated gamma distribution: A generalization of exponentiated gamma probability distribution. Applied Mathematical Sciences. 2014;8:1297–1310.
DOI: 10.12988/ams.2014. 42105.

[31] Elbatal I, Asha G, Raja AV. Transmuted exponentiated Fréchet distribution: Properties and applications. Journal of Statistics Applications and Probability. 2014;3:379–394.
DOI: 10.12785/jsap/030309.

[32] Ahmad K, Ahmad SP, Ahmed A. Structural properties of transmuted Weibull distribution. Journal of Modern Applied Statistical Methods. 2015;14:141–158.
DOI: 10.22237/ jmasm/ 1446351120.

[33] Lucena SEF, Silva AHA, Cordeiro GM. The transmuted generalized gamma distribution: Properties and application. Journal of Data Science. 2015;13:409–420.

[34] Owoloko EA, Oguntunde PE, Adejumo AO. Performance rating of the transmuted exponential distribution: An analytical approach. Springer Plus. 2015;4:8–18.
DOI: 10.1186/s40064–015–1590–6.

[35] Abdul-Moniem IB. Transmuted burr type-III distribution. Journal of Statistics: Advances in Theory and Applications. 2015;14:37–47.

[36] Abdul-Moniem IB, Seham M. Transmuted gompertz distribution. Computational and Applied Mathematics Journal. 2015;1: 88–96.

[37] Khan MS, King R. Transmuted modified inverse rayleigh distribution. Austrian Journal of Statistics. 2015; 44:17–29.
DOI:10.17713/ajs.v44i3.21.

[38] Mansour MM, Elrazik EMB, Hamed MS, Mohamed SM. A new transmuted additive Weibull distribution: Based on a new method for adding a parameter to a family of distribution. International Journal of Applied Mathematical Sciences. 2015;8:31–54.

[39] Afify AZ, Hamedani GG, Ghosh I, Mead ME. The transmuted marshall-olkin Fréchet distribution: Properties and applications. International Journal of Statistics and Probability. 2015a;4:132–148. 6.

[40] Afify AZ, Nofal ZM, Yousof HM, El-Gebaly YM, Butt NS. The transmuted Weibull-lomax distribution: Properties and application. Pakistan Journal Statistical Operation and Research. 2015b;11:135–152.
DOI: 10.18187/pjsor.v11i1.956.

[41] Iriarte YA, Astorga JM. A version of transmuted generalized rayleigh distribution. Revista Integracio´n. 2015;33:83–95.

[42] Granzotto DCT, Louzada F. The transmuted log-logistic distribution: Modeling, inference and an application to a polled tabapua race time up to first calving data. Communications in Statistics Theory and Methods. 2015;44:3387–3402.
DOI: 10.1080/03610926.2013.775307.

[43] Elbatal I, Aryal G. Transmuted dagum distribution with applications. Chilean Journal of Statistics. 2015;6:31–45.

[44] Luguterah A, Nasiru S. Transmuted exponential pareto distribution. Far East Journal of Theoretical Statistics. 2015;50:31–49.

[45] Fatima A, Roohi A. Transmuted exponentiated pareto-i distribution. Pakistan Journal Statistical Operation and Research. 2015;32:63–80.

[46] Mansour MM, Mohamed SM. A new generalized of transmuted lindley distribution. Applied Mathematical Sciences. 2015;9:2729–2748.
DOI: 10.12988/ams.2015.52158.

[47] Khan MS, King R, Hudson IL. A new three parameter transmuted chen lifetime distribution with application. Journal of Applied Statistical Sciences. 2015;21:239–259.

[48] Khan MS, King R, Hudson IL. Transmuted Kumaraswamy Distribution. Statistics in Transition. 2016b;17:1–28.
DOI: 10.21307/stattrans–2016–013.

[49] Elgarhy M, Rashed M, Shawki AW. Transmuted generalized lindley distribution. International Journal of Mathematics Trends and Technology. 2016;29:145–154.
DOI: 10.14445/22315373/IJMTT– V29P520.

[50] Vardhan RV, Balaswamy S. Transmuted new modified Weibull distribution. Mathematical Sciences and Applications E-Notes. 2016;4:125–135.

[51] Afify AZ, Yousof HM, Butt NS, Hamedani GG. The transmuted Weibull-pareto distribution. Pakistan Journal Statistical Operation and Research. 2016;32:183–206.

[52] Shahzad MN, Asghar Z. Transmuted dagum distribution: A more flexible and broad shaped hazard

function model. Hacettepe Journal of Mathematics and Statistics. 2016;45:227,244.

[53] Khan MS, King R, Hudson IL. Transmuted gompertz distribution: Properties and estimation. Pakistan Journal Statistical Operation and Research. 2016a;32:161–182.

[54] Haq MA, Butt NS, Usman RM, Fattah AA. Transmuted power function distribution. Gazi University Journal of Science. 2016;29:177–185.

[55] Chakraborty S, Bhati D. Transmuted geometric distribution with applications in modeling and regression analysis of count data. Statistics and Operations Research Transactions. 2016;40:153–176.

[56] Bourguignon M, Lea˜o J, Leiva V, Santos-Neto M. The transmuted birnbaum-saunders distribution. REVSTAT - Statistical Journal. 2017;15:601–628.

[57] Khan MS, King R, Hudson IL. Transmuted generalized exponential distribution: A generalization of the exponential distribution with applications to survival data. Communications in Statistics Simulation and Computation. 2017a;46:4377–4398.
DOI: 10.1080/03610918.2015.1118503.

[58] Cordeiro GM, Saboor A, Khan MN, Provost SB, Ortega EMM. The transmuted generalized modified Weibull distribution. Filomat. 2017;31:1395–1412.
DOI: 10.2298/FIL1705395C.

[59] Elgarhy M, Elbatal I, Diab LS, Hwas HK, Shawki AW. Transmuted generalized quasi lindley distribution. International Journal of Scientific Engineering and Science. 2017;1:1-8.

[60] Khan MS, King R, Hudson IL. Transmuted New Generalized Inverse Weibull Distribution; 2017b.

[61] Khan MS, King R, Hudson IL. Transmuted Weibull distribution: Properties and estimation. Communications in Statistics - Theory and Methods. 2017c;46:5394–5418.
DOI: 10.1080/03610926.2015.1100744.

[62] Al-babtain A, Fattah AA, Ahmed AN, Merovci F. The Kumaraswamy-transmuted exponentiated modified Weibull distribution. Communications in Statistics Simulation and Computation. 2017;46:5,3812-3832.
DOI: 10.1080/03610918.2015.1011338.

[63] Al-babtain A, Fattah AA, Ahmed AN, Merovci F. The Kumaraswamy-transmuted exponentiated modified Weibull distribution. Communications in Statistics Simulation and Computation. 2017;46:5,3812-3832.
DOI: 10.1080/03610918.2015.1011338.

[64] Al-babtain A, Fattah AA, Ahmed AN, Merovci F. The Kumaraswamy-transmuted exponentiated modified Weibull distribution. Communications in Statistics Simulation and Computation. 2017;46:5,3812-3832.
DOI: 10.1080/03610918.2015.1011338.

[65] Al-Omari AI, Al-khazaleh AMH, Alzoubi LM. Transmuted janardan distribution: A generalization of the janardan distribution. Journal of Statistics Applications and Probability. 2017;2:1-11.
DOI: 10.18576/jsap/060202.

[66] Jayakumar K, Babu MG. T-Transmuted X Family of Distributions. Statistica; 2017.
DOI: doi.org/10.6092/issn.1973-2201/6800.

[67] Venegas O, Iriarte YA, Astorga JM, Borger A, Bolfarine A, Gomez HW. A new generalization of the Maxwell distribution. Applied Mathematics and Information Sciences. 2017;3:867-876.
DOI: doi.org/10.18576/amis/110327.

[68] Nofal ZM, Afify AZ, Yousof HM, Granzotto DCT, Louzada F. The transmuted exponentiated additive Weibull distribution: Properties and applications. Journal of Modern Applied Statistical Methods. 2018;17(1): eP2526.

DOI: 10.22237/ jmasm/ 1525133340.

[69] Pobočíková I, Sedliačková Z, Michalková M. Transmuted Weibull distribution and its applications. MATEC Web of Conferences. 2018;157:1–11.
DOI: 10.1051/ matecconf/ 201815708007.

[70] Arshad MZ, Iqbal MZ, Ahmad M. Transmuted Exponentiated Moment Pareto Distribution. Open Journal of Statistics, 2018. 2017;8:939-961.

[71] Khan MS, King R, Hudson IL. Transmuted modified Weibull distribution: Properties and application. European Journal of Pure and Applied Mathematics. 2018;11:362–374.

[72] Abdullahi UK, Ieren TG. On the inferences and applications of transmuted exponential lomax distribution. International Journal of Advanced Statistics and Probability. 2018;6:30–36.
DOI: 10.14419/ijasp.v6i1.8129.

[73] Elgarhy M, Elbatal I, Haq MAU, Hassan AS. Transmuted Kumaraswamy quasi lindley distribution with applications. Annals of Data Science; 2018. DOI: 10.1007/s40745-018-0153-4.

[74] Balaswamy S. Transmuted half normal distribution. International Journal of Scientific Research in Mathematical and Statistical Sciences. 2018;5:163–170.

[75] Haq MAU, Elgarhy M, Hashmi S, Ozel G, Ain QU. Transmuted Weibull power function distribution: Its properties and applications. Journal of Data Science. 2018;397-418.
DOI: 10.6339/JDS.20180416(2).0009.

[76] Tahir A, Akhter AS, Haq MAU. Transmuted new Weibull-pareto distribution and its applications. Applications and Applied Mathematics. 2018;13:30-46.

[77] Khan MS. Transmuted generalized inverted exponential distribution with application to reliability data. Thailand Statistician. 2018;16(1):14-25.

[78] Afify AZ, Cordeiro GM, Bourguignon M, Ortega EMM. Properties of the transmuted burr XII distribution, regression and its applications. Journal of Data Science. 2018;485-510.
DOI: 10.6339/JDS.20180716(3).0003.

[79] Okorie IE, Akpanta AC. A Note on the transmuted generalized inverted exponential distribution with application to reliability data. Thailand Statistician. 2019;17:118–124.

[80] Abayomi A. Transmuted half normal distribution: Properties and application. Mathematical Theory and Modeling. 2019;9:14–26.

[81] Khan MS, King R, Hudson IL. Transmuted Burr Type X Distribution with Covariates Regression Modeling to Analyze Reliability Data. American Journal of Mathematical and Management Sciences. 2019.
DOI: 10.1080/01966324.2019.1605320.

[82] Gharaibeh MM, Al-Omari AI. Transmuted ishita distribution and its applications. Journal of Statistics Applications and Probability. 2019;8:67–81.

[83] Otiniano CEG, de Paiva BS, Daniele SB, Neto M. The Transmuted Generalized Extreme Value Distribution: Properties and Application. Communications for Statistical Applications and Methods. 2019;26:239–259.

[84] Samuel AF. On the performance of transmuted logistic distribution: Statistical properties and application. Budapest International Research in Exact Sciences (BirEx) Journal. 2019;1:26–34.

[85] Khan MS. Transmuted modified inverse Weibull distribution: Properties and application. Pakistan Journal Statistical Operation and Research. 2019;15:667-677.

[86] Khan MS, King R, Hudson IL. Transmuted exponentiated Weibull distribution. Journal of Applied Probability and Statistics. 2019;14:37-51.

[87] Ishaq AI, Usman A, Usman AU. On the generalized transmuted - rayleigh distribution. Benin Journal of Statistics. 2019;1:75-86.

[88] Riffi MI, Ansari SI, Hamdan MS. A generalized transmuted Fréchet distribution. Journal of Statistics Applications and Probability. 2019;2:1-10.

[89] Menezes A, Mazucheli J, Cardoso J, Chakraborty S. The transmuted half-normal distribution with application to precipitation data. Pesquisa Operacional. 2020;40:1-30.

[90] Aijaz A, Ahmad A, Tripathi R. Transmuted inverse lindley distribution: Statistical properties and applications. Science. Technology and Development. 2020;9:1-10.

[91] Yadav AS, Kumar D, Kumar P, Singh SK, Singh U. A new transmuted lifetime distribution: Statistical properties and application to survival data. Journal of Scientific Research; 2020.
DOI: 10.37398/JSR.2020.640133.

[92] Sarabia JM, Raja AV, Asha G. Bivariate distributions with transmuted conditionals: Models and applications. Communications in Statistics Theory and Methods. 2020;49:221–242.

[93] Badr MM, Elbatal I, Jamal F, Chesneau C, Elgarhy M. The transmuted odd Fréchet-G family of distributions: Theory and applications. Mathematics. 2020.
DOI: 10.3390/math8060958.

[94] Rahman MM, Al-Zahrani B, Shahbaz SH, Shahbaz MQ. Transmuted probability distributions: A review. Pakistan Journal of Statistics and Operation Research. 2020;16(1):83-94.

[95] Rahman MM, Al-Zahrani B, Shahbaz MQ. New general transmuted family of distributions with applications. Pakistan Journal Statistical Operation and Research. 2018c;14:807829.
DOI: 10.18187/pjsor.v14i4.2655.

[96] Rahman MM, Al-Zahrani B, Shahbaz SH, Shahbaz MQ. Cubic transmuted uniform distribution: An alternative to beta and Kumaraswamy distributions. European Journal of Pure and Applied Mathematics. 2019b;12:1106–1121.

[97] AL-Kadim KA, Mohammed MH. The cubic transmuted Weibull distribution. Journal of University of Babylon. 2017;3:862–876.

[98] Aslam M, Hussain Z, Asghar Z. Cubic Transmuted-G Family of Distributions and Its Properties. Stochastics and Quality Control; 2018.
DOI: 10.1515/eqc–2017–0027.

[99] Rahman MM, Al-Zahrani B, Shahbaz MQ. Cubic transmuted pareto distribution. Annals of Data Science. 2018b.
DOI: 10.1007/s40745–018–0178–8.

[100] Ansari SI, Eledum H. An extension of pareto distribution. Journal of Statistics Applications and Probability. 2018;7:443–455.

[101] Celik N. Some cubic rank transmuted distributions. Journal of Applied Mathematics, Statistic and Informatics. 2018;14:27–43.

[102] Saraçoğlu B, Tanış C. A new statistical distribution: Cubic rank transmuted Kumaraswamy distribution and its properties. Journal of the National Science Foundation of Sri Lanka. 2018;46(4):505-518.

[103] Riffi MI, Hamdan MS. Cubic transmuted gompertz-makeham distribution. European Journal of Advances in Engineering and Technology. 2018;5(12):1001-1010.

[104] Ansari SI, Samuh M, Bazyari A. Cubic transmuted power function distribution. Gazi University Journal of Science. 2019.
DOI: 10.35378/gujs.470682.

[105] Rahman MM, Al-Zahrani B, Shahbaz MQ. Cubic Transmuted Weibull distribution: Properties and applications. Annals of Data Science; 2019a.

[106] Bhatti FA, Hamedani GG, Sheng W, Ahmad M. Cubic rank transmuted modified burr III pareto distribution: Development, properties, characterizations and applications. International Journal of Statistics and Probability. 2019;8:94–112.
DOI: 10.5539/ijsp.v8n1p94.

[107] Adeyinka FS. On Modelling of infant mortality rate in Nigeria with exponentiated cubic transmuted exponential distribution. International Journal on Data Science and Technology. 2020;6(1):16-22.

[108] Eledum H. Some cubic transmuted exponentiated pareto-1 distribution. Journal of Mathematics and Statistics. 2020;16:113.124.

[109] Ogunde AA, Olayode F, Adejumoke A. Cubic transmuted gompertz distribution: As a life time distribution. Journal of Advances in Mathematics and Computer Science. 2020;35(1):sx105-116.

[110] Akter S, Khan MAI, Rana MS, Rahman MM. Cubic transmuted burr-XII distribution with properties and applications. Journal of Statistics Applications and Probability Letters. 2021;1:23-32.

[111] Ali MA, Athar H. Generalized rank mapped transmuted distribution for generating families of continuous distributions. Journal of Statistical Theory and Applications; 2021.
DOI: doi.org/10.2991/jsta.d.210129.001.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sdiarticle4.com/review-history/69356