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Hyperstability of a Monomial Functional Equation

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Abstract

In this paper, we establish some hyperstability results concerning the monomial functional equation

$$\sum_{r=0}^{n}(-1)^{n-r}C_r^n f(rx+y) = n!f(x)$$

in Banach spaces.

Keywords: Hyperstability, monomial functional equation, fixed point theorem.

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1 Introduction

The starting point of studying the stability of functional equations seems to be the famous talk of Ulam [1],[2] in 1940, in which he discussed a number of important unsolved problems. Among those was the question concerning the stability of group homomorphisms.

Let G_1 be a group and let G_2 be a metric group with a metric $d(.,.)$. Given $\varepsilon > 0$, does there exists a $\delta > 0$ such that if a mapping $h : G_1 \to G_2$ satisfies the inequality $d\Big(h(xy), h(x)h(y)\Big) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d\Big(h(x), H(x)\Big) < \varepsilon$ for all $x \in G_1$?

The case of approximately additive mapping was solved by Hyers [3] under the assumption that G_1 and G_2 are Banach spaces. Later, the result of Hyers was significantly generalized by Rassias [4] and Găvruta [5]. Since then, the stability problems of several functional equations have been extensively investigated.

Let X and Y be linear spaces and Y^X be the vector space of all functions from X to Y. Following [6], for each $x \in X$, define $\Delta_x : Y^X \to Y^X$ by

$$\Delta_x f(y) = f(x+y) - f(y) \quad (f \in Y^X, y \in X).$$

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Inductively, we define

$$\Delta_{x_1,...,x_n} f(y) = \Delta_{x_1,...,x_{n-1}}(\Delta_{x_n} f(y))$$

for each $y, x_1, ..., x_n \in X$ and all $f \in Y^X$. If $x_1 = ... = x_n = x$, we write

$$\Delta_x^n f(y) = \Delta_{\underbrace{x, ..., x}_{n\ times}} f(y)$$

By induction on n, it can be easily verified that

$$\Delta_x^n f(y) = \sum_{r=0}^{n} (-1)^{n-r} C_r^n f(rx + y) \quad (n \in \mathbb{N}, x, y \in X), \tag{1}$$

where $C_r^n = \frac{n!}{(n-r)!r!}$.

The functional equation

$$\sum_{r=0}^{n} (-1)^{n-r} C_r^n f(rx + y) = n! f(x) \tag{2}$$

is called *the monomial functional equation* of degree n, since the function $f(x) = cx^n$ is a solution of this functional equation. Every solution of the monomial functional equation of degree n is said to be *a monomial mapping* of degree n. In particular additive, quadratic, cubic and quartic functions are monomials of degree one, two, three and four respectively. The stability of monomial equations was initiated by Hyers in [6]. The problem has been recently considered by many authors, we refer, for example, to [7], [8], [9], [10] and [11].

Kaiser [12] proved the stability of monomial functional equation where the functions map a normed space over a field with valuation to a Banach space over a field with valuation and the control function is of the form $\varepsilon\Big(\|x\|^\alpha + \|y\|^\alpha\Big)$. In 2007, Cădariu and Radu [13] proved the stability of the monomial functional equation

$$\sum_{i=0}^{n} C_i^n (-1)^{n-i} f(ix + y) - n! f(x) = 0 \tag{3}$$

In 2008, Lee [14] modified the results of Cădarui and Radu for the stability of the monomial functional equation (3) in the sense of Rassias and he investigated the superstability of this equation. In 2010, Mirmostafaee [15] proved the Hyers-Ulam stability of monomial functional equation of an arbitrary degree in non-Archimedean normed spaces over a field with valuation and the control function $\varphi(x, y)$.

In 2014, Almahalebi, Sirouni, Charifi and Kabbaj [16] proved the fuzzy stability of the monomial functional equation with the control function is of the form $N'\Big(\varphi(x, y), t\Big)$ where N' is a fuzzy norm.

We say a functional equation $\mathfrak{D}$ is *hyperstable* if any function f satisfying the equation $\mathfrak{D}$ approximately is a true solution of $\mathfrak{D}$. It seems that the first hyperstability result was published in [17] and concerned the ring homomorphisms. However, The term *hyperstability* has been used for the first time in [18]. Quite often the hyperstability is confused with superstability, which admits also bounded functions.

Numerous papers on the hyperstability of functional equations have been published by different authors, we refer, for example, to [19], [20], [21], [22], [23], [24] and [25].

In this paper, we present the hyperstability results for the monomial functional equation (2). The method of the proofs used in the main results is based on a fixed point result that can be derived from [Theorem 1 [26]]. To present it we need the following three hypothesis:

(**H1**) X is a nonempty set, Y is a Banach space, $f_1, ..., f_k : X \longrightarrow X$ and $L_1, ..., L_k : X \longrightarrow \mathbb{R}_+$ are given.

(**H2**) $\mathcal{T} : Y^X \longrightarrow Y^X$ is an operator satisfying the inequality

$$\Big\|\mathcal{T}\xi(x) - \mathcal{T}\mu(x)\Big\| \le \sum_{i=1}^{k} L_i(x)\Big\|\xi\Big(f_i(x)\Big) - \mu\Big(f_i(x)\Big)\Big\|, \qquad \xi, \mu \in Y^X, \quad x \in X.$$

(**H3**) $\Lambda : \mathbb{R}_+^X \longrightarrow \mathbb{R}_+^X$ is a linear operator defined by

$$\Lambda\delta(x) := \sum_{i=1}^{k} L_i(x)\delta\Big(f_i(x)\Big), \qquad \delta \in \mathbb{R}_+^X, \quad x \in X.$$

The following theorem is the basic tool in this paper. We use it to assert the existence of a unique fixed point of operator $\mathcal{T} : Y^X \longrightarrow Y^X$

Theorem 1. *Let hypotheses (**H1**)-(**H3**) be valid and functions $\varepsilon : X \longrightarrow \mathbb{R}_+$ and $\varphi : X \longrightarrow Y$ fulfil the following two conditions*

$$\|\mathcal{T}\varphi(x) - \varphi(x)\| \le \varepsilon(x), \qquad x \in X,$$

$$\varepsilon^*(x) := \sum_{n=0}^{\infty} \Lambda^n \varepsilon(x) < \infty, \qquad x \in X.$$

Then there exists a unique fixed point ψ of $\mathcal{T}$ with

$$\|\varphi(x) - \psi(x)\| \le \varepsilon^*(x), \qquad x \in X.$$

Moreover

$$\psi(x) := \lim_{n\to\infty} \mathcal{T}^n\varphi(x), \qquad x \in X.$$

2 Hyperstability Results

The following theorems are the main results in this paper and concern the hyperstability of equation (2).

Let X be a normed space, Y be a Banach space, $\theta \ge 0$, $p < 0$ and let $f : X \longrightarrow Y$ satisfy

$$\left\|\sum_{r=0}^{n}(-1)^{n-r}C_r^n f(rx+y) - n!f(x)\right\| \le \theta\Big(\|x\|^p + \|y\|^p\Big) \tag{4}$$

for all $x, y \in X \setminus \{0\}$. Then f is a monomial mapping of degree n on $X \setminus \{0\}$.

Proof. Replace x by $(m+1)x$ and y by $-mx$, where $m \in \mathbb{N}$, in (4). We get that

$$\left\|f(x) - \frac{1}{n}f(-mx) + (-1)^n(n-1)!f\Big((m+1)x\Big) + \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n f\Big(\big((r-1)m+r\big)x\Big)\right\|$$
$$\le \frac{\theta}{n}\Big((m+1)^p + m^p\Big)\|x\|^p \tag{5}$$

for all $x \in X \setminus \{0\}$.

Further put

$$\mathcal{T}_m\xi(x) := \frac{1}{n}f(-mx) - (-1)^n(n-1)!f\Big((m+1)x\Big) - \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n f\Big(\big((r-1)m+r\big)x\Big),$$

and

$$\varepsilon_m(x) := \frac{\theta}{n}\Big((m+1)^p + m^p\Big)\|x\|^p$$

for all $x \in X \setminus \{0\}, \xi \in Y^{X\setminus\{0\}}$. Then, the inequality (5) takes the following form

$$\|\mathcal{T}_m f(x) - f(x)\| \le \varepsilon_m(x), \qquad x \in X \setminus \{0\}.$$

The following linear operator

$$\Lambda_m \delta(x) := \frac{1}{n}\delta(-mx) + (n-1)!\delta\Big((m+1)x\Big) + \frac{1}{n}\sum_{r=2}^{n} C_r^n \delta\Big(\big((r-1)m+r\big)x\Big), \qquad \delta \in \mathbb{R}_+^{X\setminus\{0\}}, x \in X\setminus\{0\}$$

has the form described in (**H3**) with $k = n+1$ and
$f_{n+1}(x) = -mx, \quad f_1(x) = (m+1)x, \quad f_r(x) = \big((r-1)m+r\big)x,$
$L_{n+1}(x) = \frac{1}{n}, \quad L_1(x) = (n-1)!, \quad L_r(x) = \frac{1}{n}C_r^n$ where $r = 2, 3, ..., n$, for $x \in X \setminus \{0\}$.

Moreover, for every $\xi, \mu \in Y^{X\setminus\{0\}}$, and every $x \in X \setminus \{0\}$, we have

$$\begin{aligned}\Big\|\mathcal{T}_m\xi(x) - \mathcal{T}_m\mu(x)\Big\| &= \Bigg\|\frac{1}{n}\xi(-mx) + (-1)^n(n-1)!\xi\Big((m+1)x\Big) - \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n\xi\Big(\big((r-1)m+r\big)x\Big) \\ &\quad -\frac{1}{n}\mu(-mx) - (-1)^n(n-1)!\mu\Big((m+1)x\Big) + \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n\mu\Big(\big((r-1)m+r\big)x\Big)\Bigg\| \\ &\le \frac{1}{n}\Big\|(\xi-\mu)(-mx)\Big\| + (n-1)!\Big\|(\xi-\mu)\Big((m+1)x\Big)\Big\| + \frac{1}{n}\sum_{r=2}^{n} C_r^n\Big\|(\xi-\mu)\Big(\big((r-1)m+r\big)x\Big)\Big\| \\ &= \sum_{i=1}^{n+1} L_i(x)\Big\|\xi\Big(f_i(x)\Big) - \mu\Big(f_i(x)\Big)\Big\|\end{aligned}$$

and so (**H2**) is valid. Next, we can find $m_0 \in \mathbb{N}$ such that

$$\alpha_m = \frac{m^p}{n} + (n-1)!(m+1)^p + \frac{1}{n}\sum_{r=2}^{n} C_r^n\Big((r-1)m+r\Big)^p < 1$$

for all $m \ge m_0$. Therefore, we have

$$\begin{aligned}\varepsilon_m^*(x) &:= \sum_{s=0}^{\infty}\Lambda_m^s\varepsilon_m(x) \\ &= \sum_{s=0}^{\infty}\Lambda_m^s\left(\frac{\theta}{n}\Big((m+1)^p+m^p\Big)\|x\|^p\right) \\ &= \frac{\theta}{n}\Big((m+1)^p+m^p\Big)\sum_{s=0}^{\infty}\left(\frac{m^p}{n} + (n-1)!(m+1)^p + \frac{1}{n}\sum_{r=2}^{n}C_r^n\Big((r-1)m+r\Big)^p\right)^s\|x\|^p \\ &= \frac{\theta\Big((m+1)^p+m^p\Big)\|x\|^p}{n(1-\alpha_m)}\end{aligned}$$

for all $x \in X \setminus \{0\}$ and $m \ge m_0$.

Thus, according to Theorem 1, for each $m \geq m_0$ there exists a unique solution $F_m : X\backslash\{0\} \longrightarrow Y$ of the equation

$$F_m(x) = \frac{1}{n}F_m(-mx) - (-1)^n(n-1)!F_m\Big((m+1)x\Big) - \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n F_m\Big(\big((r-1)m+r\big)x\Big)$$

such that

$$\Big\|f(x) - F_m(x)\Big\| \leq \frac{\theta\Big((m+1)^p + m^p\Big)\|x\|^p}{n(1-\alpha_m)}$$

for all $x \in X \setminus \{0\}$ and $m \geq m_0$. Moreover,

$$F_m(x) := \lim_{s\to\infty} \mathcal{T}_m^s f(x), \qquad x \in X \setminus \{0\}.$$

To prove that $F_m(x)$ satisfies (2) on $X \setminus \{0\}$, observe that

$$\left\|\sum_{r=0}^{n}(-1)^{n-r}C_r^n \mathcal{T}_m^s f(rx+y) - n!\mathcal{T}_m^s f(x)\right\| \leq \theta\alpha_m^s\Big(\|x\|^p + \|y\|^p\Big) \tag{6}$$

for all $x, y \in X \setminus \{0\}$ and all $s \in \mathbb{N}$.
Indeed, if $s = 0$, then (6) is simply (4). So, take $t \in \mathbb{N}_0$ and suppose that (6) holds for $s = t$ and all $x, y \in X \setminus \{0\}$. Then

$$\begin{aligned}
&\left\|\sum_{k=0}^{n}(-1)^{n-k}C_k^n \mathcal{T}_m^{t+1} f(kx+y) - n!\mathcal{T}_m^{t+1} f(x)\right\| = \left\|\sum_{k=0}^{n}(-1)^{n-k}C_k^n\left\{\frac{1}{n}\mathcal{T}_m^t f\Big(-m(kx+y)\Big)\right.\right.\\
&\left. -(-1)^n(n-1)!\mathcal{T}_m^t f\Big((m+1)(kx+y)\Big) - \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n \mathcal{T}_m^t f\Big(\big((r-1)m+r\big)(kx+y)\Big)\right\}\\
&\left.-n!\left\{\frac{1}{n}\mathcal{T}_m^t f(-mx) - (-1)^n(n-1)!\mathcal{T}_m^t f\Big((m+1)x\Big) - \frac{1}{n}\sum_{r=2}^{n}(-1)^{r-1}C_r^n \mathcal{T}_m^t f\Big(\big((r-1)m+r\big)x\Big)\right\}\right\|\\
&\leq \frac{1}{n}\left\|\sum_{k=0}^{n}(-1)^{n-k}C_k^n \mathcal{T}_m^t f\Big(-m(kx+y)\Big) - n!\mathcal{T}_m^t f(-mx)\right\|\\
&+(n-1)!\left\|\sum_{k=0}^{n}(-1)^{n-k}C_k^n \mathcal{T}_m^t f\Big((m+1)(kx+y)\Big) - n!\mathcal{T}_m^t f\Big((m+1)x\Big)\right\|\\
&+\frac{1}{n}\sum_{r=2}^{n}C_r^n\left\|\sum_{k=0}^{n}C_k^n \mathcal{T}_m^t f\Big(\big((r-1)m+r\big)(kx+y)\Big) - n!\mathcal{T}_m^t f\Big(\big((r-1)m+r\big)x\Big)\right\|\\
&\leq \theta\alpha_m^t\left(\frac{m^p}{n} + (n-1)!(m+1)^p + \frac{1}{n}\sum_{r=2}^{n}C_r^n\big((r-1)m+r\big)^p\right)\Big(\|x\|^p + \|y\|^p\Big)\\
&= \theta\alpha_m^{t+1}\Big(\|x\|^p + \|y\|^p\Big).
\end{aligned}$$

By induction, we shown that (6) holds for all $x, y \in X \setminus \{0\}$ and all $s \in \mathbb{N}_0$. Letting $s \longrightarrow \infty$ in (6), we show that

$$\sum_{r=0}^{n}(-1)^{n-r}C_r^n F_m(rx+y) - n!F_m(x) = 0$$

for all $x, y \in X \setminus \{0\}$. Thus, we have proved that, for every $m \geq m_0$, there exists a unique mapping $F_m : X \setminus \{0\} \to Y$ such that F_m is a monomial mapping of degree n on $X \setminus \{0\}$ and

$$\left\| f(x) - F_m(x) \right\| \leq \frac{\theta\big((m+1)^p + m^p\big)}{n(1-\alpha_m)} \|x\|^p$$

for all $x \in X \setminus \{0\}$.
Since $p < 0$, the sequence $\left\{ \frac{\theta((m+1)^p+m^p)}{n(1-\alpha_m)} \|x\|^p \right\}_{m \geq m_0}$ tends to zero when $m \longrightarrow \infty$. Consequently, f is a monomial mapping of degree n on $X \setminus \{0\}$ as the pointwise of $\{F_m\}_{m \geq m_0}$.

In a similar way we can prove the following theorem
Let X be a normed space, Y be a Banach space, $\theta \geq 0$, $p, q \in \mathbb{R}$, $p + q < 0$ and let $f : X \longrightarrow Y$ satisfy

$$\left\| \sum_{r=0}^{n} (-1)^{n-r} C_r^n f(rx + y) - n! f(x) \right\| \leq \theta \|x\|^p \|y\|^q \tag{7}$$

for all $x, y \in X \setminus \{0\}$. Then f is a monomial mapping of degree n on $X \setminus \{0\}$.

Proof. Since $p + q < 0$, one of p, q must be negative. Assume that $q < 0$ and replace y by mx where $m \in \mathbb{N}$, in (7). Then we get

$$\left\| \frac{1}{n!} \sum_{r=0}^{n} (-1)^{n-r} C_r^n f\Big((r+m)x\Big) - f(x) \right\| \leq \frac{\theta m^q}{n!} \|x\|^{p+q} \tag{8}$$

for all $x \in X \setminus \{0\}$.
Similarly, we define

$$\mathcal{T}_m \xi(x) := \frac{1}{n!} \sum_{r=0}^{n} (-1)^{n-r} C_r^n \xi\Big((r+m)x\Big), \qquad x \in X \setminus \{0\}, \quad \xi \in Y^{X \setminus \{0\}},$$

$$\varepsilon_m(x) := \frac{\theta m^q}{n!} \|x\|^{p+q}, \qquad x \in X \setminus \{0\},$$

$$\Lambda_m \delta(x) := \frac{1}{n!} \sum_{r=0}^{n} C_r^n \delta\Big((r+m)x\Big), \qquad x \in X \setminus \{0\}, \delta \in \mathbb{R}_+^{X \setminus \{0\}}.$$

and as in Theorem 2 we observe that (8) takes the form

$$\left\| \mathcal{T}_m f(x) - f(x) \right\| \leq \varepsilon_m(x), \qquad x \in X \setminus \{0\}$$

and Λ_m has the form described in (**H3**) with $k = n + 1$ where

$f_i(x) = (i - 1 + m)x$ and $L_i(x) = \frac{1}{n!} C_{i-1}^n$ where $i = 1, ..., n+1$ for $x \in X \setminus \{0\}$.
Moreover, for every $\xi, \mu \in Y^{X \setminus \{0\}}$, $x \in X \setminus \{0\}$ we have

$$\left\| \mathcal{T}_m \xi(x) - \mathcal{T}_m \mu(x) \right\| \leq \sum_{i=1}^{n+1} L_i(x) \left\| (\xi - \mu)\big(f_i(x)\big) \right\|,$$

and so (**H2**) is valid. Next we can find $m_0 \in \mathbb{N}$ such that

$$\alpha_m = \frac{1}{n!} \sum_{i=0}^{n} C_r^n (r+m)^{p+q} < 1$$

for all $m \geq m_0$. Therefore

$$\begin{aligned}
\varepsilon_m^*(x) &:= \sum_{s=0}^{\infty} \Lambda_m^s \varepsilon_m(x) \\
&= \sum_{s=0}^{\infty} \Lambda_m^s \left(\frac{\theta m^q}{n!} \|x\|^{p+q} \right) \\
&= \frac{\theta m^q}{n!} \sum_{s=0}^{\infty} \left(\frac{1}{n!} \sum_{r=0}^{n} C_r^n \|(r+m)x\|^{p+q} \right)^s \\
&= \frac{\theta m^q}{n!} \|x\|^{p+q} \sum_{s=0}^{\infty} \left(\frac{1}{n!} \sum_{r=0}^{n} C_r^n (r+m)^{p+q} \right)^s \\
&= \frac{\theta m^q}{n!(1-\alpha_m)} \|x\|^{p+q}
\end{aligned}$$

for all $x \in X \setminus \{0\}$ and $m \geq m_0$.
Hence, according to Theorem 1, for each $m \geq m_0$ there exists a unique solution $F_m : X \setminus \{0\} \longrightarrow Y$ of the equation

$$F_m(x) = \frac{1}{n!} \sum_{r=0}^{n} (-1)^{n-r} C_r^n F_m\Big((r+m)x\Big), \qquad x \in X \setminus \{0\}$$

such that

$$\Big\| f(x) - F_m(x) \Big\| \leq \frac{\theta m^q \|x\|^{p+q}}{n!(1-\alpha_m)}, \qquad x \in X \setminus \{0\}, m \geq m_0.$$

Moreover,

$$\sum_{r=0}^{n} (-1)^{n-r} C_r^n F_m(rx+y) = n! F_m(x), \qquad x \in X \setminus \{0\}.$$

In this way we obtain a sequence $\{F_m\}_{m \geq m_0}$ of monomial mappings on $X \setminus \{0\}$ such that

$$\Big\| f(x) - F_m(x) \Big\| \leq \frac{\theta m^q \|x\|^{p+q}}{n!(1-\alpha_m)}, \qquad x \in X \setminus \{0\}, m \geq m_0.$$

It follows, with $m \longrightarrow \infty$, that f is a monomial mapping of degree n on $X \setminus \{0\}$.

□

3 Conclusion

This paper indeed presents a relationship between three various disciplines: the theory of Banach spaces, the theory of stability of functional equations and the fixed point theory. We established some hyperstability results concerning a monomial functional equation in Banach spaces by using fixed point theorem which given by Brzdęk J. Chudziak J. and Páles Zs. [26].

Competing Interests

The authors declare that no competing interests exist.

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