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Graph Partitioning Based Normalized Cut Methods

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Method Article

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Abstract

The process of image segmentation is one of the most important steps in computer vision for image retrieval, visual summary, image based modeling and in many other processes. The goal of segmentation is typically to locate certain objects of interest. In this paper, we have studied and investigated graph based normalized cut segmentation methods and proposed a technique for adding flexibility to the parameters for performance improvement. These methods are examined analytically and tested their performance for the standard images. The results obtained for the important metrics show that these methods perform better than others approaches and are computationally efficient, and useful for precise image segmentation.

Keywords: Normalized cut, two-way cut, k-way cut, watershed based regions.

1 Introduction

Most Commonly used segmentation methods are based on clustering, compression, histogram, edge detection, region growing, partial differential equations, and graphs [1]. Among these segmentation methods, the graph based segmentation approach has attracted many attentions and become one of the most successful research areas in computer vision in recent years. In these

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methods, the set of points in an arbitrary feature space are represented by weighted undirected graph, $G = (V, E)$, where V is the set of nodes called pixels and an edge set E contains edges formed by joining every pair of nodes. Weight of each edge $w(i, j)$ is function of similarity between nodes V_i and V_j. The set of nodes are partitioned into disjoint sets $V_{1,}\ V_{2,}\ V_{3,}\ \ldots\ V_n$ such that the nodes in V_i has strong affinities between them.

Partitioning to achieve better segmentation poses several challenges such as the precise criteria for good partition and its efficient computation. Most of the graph based methods are based on local properties of the graph. The partitioning criterion discussed above fails to extract global impressions of the scene. This paper focuses on exploring of graph based segmentation using normalized cut method. The rest of the paper is organized as follows. Section II discusses the normalized cut method and its variations. Section III presents proposed technique and its simulation results showing the performance of various methods discussed and finally conclude in Section IV. Normalized cut is used for extracting global impression of an image than focusing on local features and their consistencies in the image data. It deals with both the total dissimilarities between the different partitions as well as the total similarities within the same partition.

2 Normalized Cut Method

Any graph $G = (V, E)$ can be partitioned into two disjoint sets $A,\ B$ provided that $|V|$ is greater than 1. The degree of dissimilarity between the sets A and B is sum of all the weights of edges between nodes in A to nodes in B called as cut value,

$$Cut(A, B) = \sum_{u \in A, v \in B} w(u, v) \qquad (\mathbf{1})$$

The optimal bi-partitioning of a graph is the one that minimizes cut value. By considering every possible partition, minimum cut for a graph can be obtained, but it is very complex problem. Finding minimum cut is well studied problem and there exists efficient algorithms for solving it. Wu et al. [2] proposed a clustering method based on minimum cut criterion. In this method, graph is partitioned into k – subgraphs such that the maximum cut across subgraphs is minimized. This problem can be efficiently solved by recursively finding the minimum cuts that bisect the existing segments. However, this criterion is suitable for cutting of small sets of isolated nodes in the graph, and can give inaccurate segmentation. This is because by using (1), cut value increases if the numbers of crossings between the two partitioned segments are more. If two partitions are equally sized, they will be related by more edges than the unequally sized partitions. In Fig. 1(a), A and B have 8 vertices in each partition and 64 edge crossing between the partitions whereas, in Fig. 1(b), A has 15 vertices and B has only one vertex and only 15 edge crossing between the partition. To avoid this unnatural bias for partitioning, Shi *et al.* proposed a new measure of disassociation, the normalized cut *Ncut* [3], which is obtained by taking the sum of the ratio of cut value of the partition and total of weights of nodes in each partition. The best normalized cut in a graph is one which minimizes the *Ncut* value. For solving this minimization problem, second smallest eigen vector of the generalized eigen system $(D - W)y = \lambda y$ can be used, where D is $n\,X\,n$ diagonal matrix with degrees d on the diagonal and W be $n\,X\,n$ symmetric matrix with $W_{ij} = w(i, j)$ called as affinity matrix. In affinity matrix, nodes with more similarities will have high weight values whereas nodes with less similarities will have low weight values. There are various types of normalized cut methods, namely recursive two way- cut, simultaneous K-way cut, pixel affinity, multi-scale graph decomposition, and watershed regions based similarity graph.

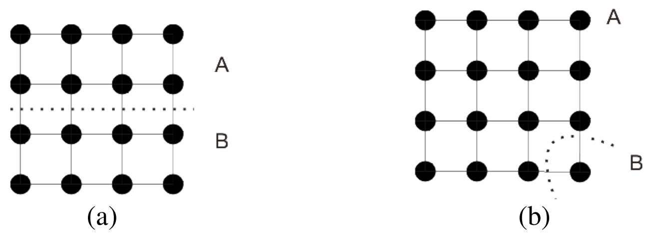


Fig. 1. Graphical representation of 4 x 4 pixel image (a) Two equally sized partitions A and B (b) Two unequally sized partitions A and B

2.1 Recursive Two Way Cut

To split graph, the graph nodes are partitioned into two subsets using threshold value. The cut can recursively be obtained in two partitioned parts and stops when it reaches to previously given *Ncut* value. This technique is known as Recursive Two Way Cut [4] and follows the steps:

i. For given weighted graph *G,* summarize the information into diagonal matrix *D* and the affinity matrix *W.* Define weight w_{ij} of an edge connecting to two nodes *i* and *j* by using brightness value of the pixel and their spatial location as

$$w_{ij} = e^{\frac{-\|F(i)-F(j)\|_2^2}{\sigma_F}} \cdot \begin{cases} e^{\frac{-\|X_i - X_i\|_2}{\sigma_X}}, & if\ \|X_i - X_i\|_2 < r \\ 0, & otherwise \end{cases} \quad \ldots\ldots(\mathbf{2})$$

where X_i is the spatial location of node i.
$F(i)$ is a feature vector based on intensity, color or texture of node i.
σ_F and σ_X are spatial tuning parameters.
w_{ij} is an entry in affinity matrix *W.*

ii. Solve $(D - W)y = \lambda D_y$ for eigen vectors with the smallest eigen value.

iii. Use eigen vector corresponding to the second smallest eigen value to bipartition the graph by finding the splitting point so that the *Ncut* can be minimized.

iv. Repeat steps for each subgraph if *Ncut* in Step ***iii*** is below the prespecified threshold.

v. If number of segments is specified by the user then recursively repartition the segmented part.

The number of graphs segmented by this method is controlled directly by the maximum allowed *Ncut.*

2.2 Simultaneous K Way Cut:

Instead of using second smallest eigen vector to partition the graph, K-way partition can be obtained by using all top eigen vectors [5]. To achieve this, one can use simple clustering algorithm to over segment image into k groups. Then either iteratively merge two segments simultaneously until only k segments are left or from the k groups obtained in first step build a condensed graph and recursively bipartion the graph using *Ncut* criterion.

2.3 Pixel Affinity Graph

In this method, each pixel is taken as graph node and two pixels within *r* distance are connected by an edge. Similarity between the connected pixels reflects weight of an edge. The grouping cue used in the similarity pixel will reflect the overall quality of segmentation such as intensity positions and contours [6]. The measure of similarity, W_{ip} for grouping cue is obtained by considering position and the intensity difference between pixels i and *j*, graph connection radius, scale parameters which controls the tradeoff between brightness similarity and spatial proximity. When this grouping cue is used separately, it often results into inaccurate segmentation since some natural images are affected by texture disorder. Hence another grouping cue, W_{ic} related to the intervening contours is calculated by using straight line joining pixels *i* and *j,* and square of edge strength at fixed location. If straight line across pixel does not cross an image edge then they have high affinity. These two grouping cue can be combined as

$$W_{ipc}(i,j) = \sqrt{W_{ip}(i,j).W_{ic}(i,j)} + \alpha\, W_{ic}(i,j) \qquad \dots\dots(\mathbf{3})$$

where α is constant. For objects with larger radius *r* with weak contours can be detected more easily, however the graph affinity matrix becomes denser. For larger graph radius, generally the segmentation quality is better, but speed is very slow. Across larger image regions, long range graph connections facilitate propagation of local grouping cues. In such situations, objects with weak contours in cluttered background can be detected easily.

2.4 Multi-scale Graph Decomposition

To collect sufficient grouping information, affinity graph needs long range connections. These connections can be compressed on a multi-scale grid. It can produce precise object boundaries with constrained segmentation. Sharon et al. [7] used algebraic multi-grid method to solve normalized cut criterion efficiently and effective graph coarsening is used for developing irregular pyramid encoding region based grouping cues. Benzit et al. [8] proposed the decomposition of multiple scales, in which graph links can be separated into different scales. The constrained normalized cut is given by

$$Maximize\ \epsilon\ (X) = \frac{1}{K}\sum_{l=1}^{k}\frac{X_l^T\ W\ X_l}{X_l^T D\ X_l} \qquad \dots\dots(\mathbf{4})$$

This algorithm works simultaneously across the graph scales, with an inter-scale constraint to ensure communication and consistency between the segmentation at each scale. This method for segmentation is computationally efficient and suitable for segmentation of large images.

2.5 Watershed Regions Based Similarity Graph

Watershed transformation is a morphological based tool for image segmentation. The watershed transform can be classified as a region-based segmentation approach. In hierarchical watershed approach, local minima are chosen as region seeds. It adds neighbors to priority queue, sorted by value then take top priority pixel from queue, and repeat the process until it is finished. The flooding process starts with given threshold value t_v that represents some relief features. Hence, some initial regions will be flooded which yields desired number of partitions. The hierarchical watershed regions can be modeled using graph [9]. The flooded gradient image is represented by connected weighted neighbourhood graph, where node is the catchment basin of the topographic surface.

3 Results and Discussion

The N*cut* algorithm first reads an image of size $n\,X\,n$ and constructs an intensity matrix corresponding to the pixels in an image where intensity matrix consists of feature values or the intensity values of the pixel. Then the graph function computes the affinity matrix of an image by setting default values to the parameters as $\sigma_F = 0.1, \sigma_X = 0.3\ and\ r = 10$. Parameter σ_F is tuning parameter which controls magnitude of the difference features involved in computing w_{ij}. From (2), we can see that if the value of σ_F is smaller, then the weight w_{ij} is less which results into closely grouped pixels and more local segmentation and vice versa. Similarly the parameter σ_X is tuning parameter which controls degree of the spatial features also involved in computing w_{ij}. But in this algorithm, parameters σ_F and σ_X are constant, and applied to all features in the image despite the consequences of the feature values. As a result N*cut* algorithm achieves global segmentation which is not perceptive to local variations in the image. Hence, by adding flexibility to the parameter values possessing fixed values in original N*cut,* we can improve its performance by constructing the affinity matrix through local tuning. We correlated the features values around pixel i and j by modeling σ_F as $[\sigma(F(i),r).\sigma(F(j),r)]^{-1}$ where $\sigma(F(i),r)$ and $\sigma(F(j),r)$ are the standard deviations of neighborhood features around pixel *i* and pixel *j* respectively, around radius *r*. σ_F will capture the correlation of features between pixel *i* and pixel *j* while determining the weights of edges. For fixed radius, local variations in pixel *i* will be less for smaller values of $\sigma(F(i),r)$. In the same manner, features around *j* will be less for smaller values of $\sigma(F(j),r)$. For low variations in combined local features around pixel *i* and pixel *j*, $\sigma(F(i),r).\sigma(F(j),r)$ will also be smaller, resulting better segmentation quality with linear complexity. Because of lack of well established image segmentation quality metric and to compare the results to ground truth boundaries, we need to threshold boundary maps multiple times at each level by using Precision (P) and Recall (R). Precision is the probability that machine generated boundary pixel is true boundary pixel. Recall is the probability that the border pixel marked by the machine is same as the border pixel marked by human. Harmonic mean of precision and recall can be summarized in terms of F-measure [10]. For performance analysis, standard images from the Berkeley Segmentation Dataset (BSDS) and Benchmark [11] are used. Precision, recall, and F-measure are calculated for each segmented image as shown in Fig. 2. The results obtained with flexibility of parameter values show improvement and are better than combining hierarchical multiscale graph decomposition reported in [12].

F-measure values shown in Fig. 3(a) for the selected images demonstrate that multi-scale decomposition performs well for these images. Significant enhancement in F-measure by using proposed technique illustrates better quality of image segmentation as compare to the original N*cut*.

Fig. 3(b) shows the time complexity for various methods for selected images. The results illustrates that the multi-scale decomposition and watershed region based methods have least time requirements and can perform well for complex images. The computational time required in case of proposed technique is lesser than that of original N*cut*.

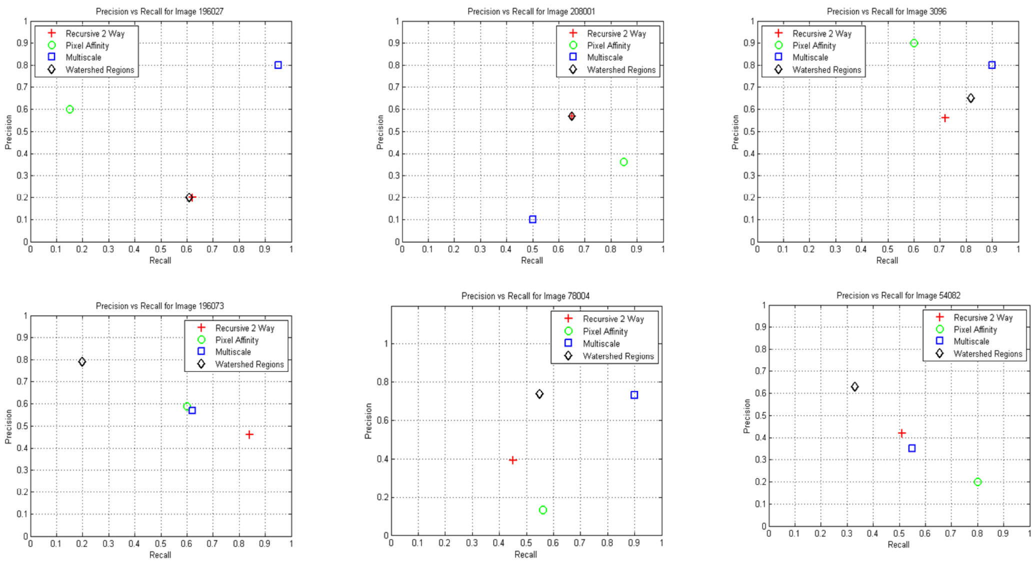
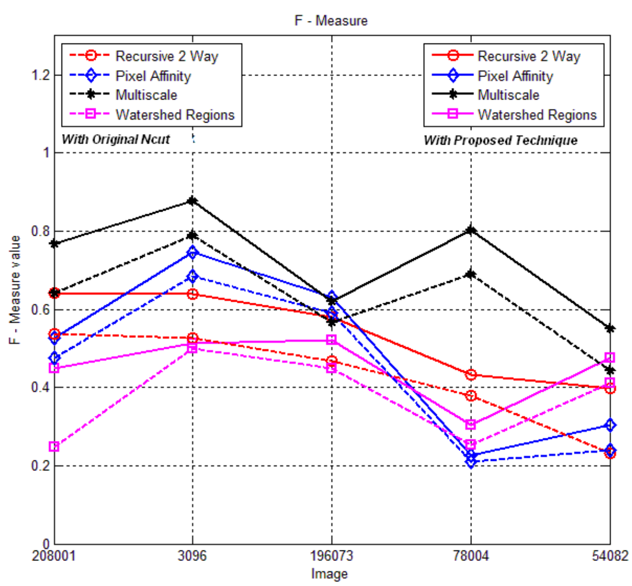
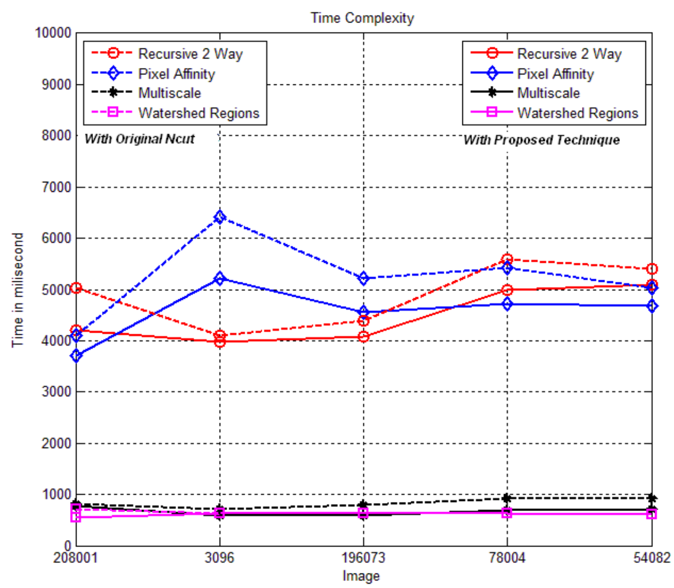


Fig. 2. Precision Recall for various images of BSDS (196027, 208001, 3096, 196073, 78004, 4082)



(a) F-Measure



(b) Time Complexity

Fig. 3. Metrics for various images of BSDS (208001, 3096, 196073, 78004, 54082, 196027)

4 Conclusion

This paper presents various graph based normalized cut segmentation methods and their variations such as recursive two way-cut, simultaneous K-way cut, pixel affinity, multi-scale graph decomposition, and watershed regions approach. These methods are studied analytically and tested their performance for the standard images. We have proposed a technique to add flexibility to the parameter values possessing fixed values in original normalized cut technique so that local as well as global features can be extracted. It establishes strong weight connections between the identical neighboring pixels in the affinity matrix resulting better segmentation quality with linear complexity. The result obtained shows that these methods can perform better than original N*cut*, and has less time complexity.

Herein, we present the hybrid of a new conjugate gradient method and Galerkin theory to engineers and scientists who wish to solve real life problems in this class of boundary value problems.

Competing Interests

Authors have declared that no competing interests exist.

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